Distance Distributivity in Polish: A Glue Semantics Approach

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We propose a novel syntactico-semantic analysis of distance distributivity in Polish and other languages, which is couched in Lexical Functional Grammar coupled with Glue Semantics. We introduce and analyse a troublesome construction, apparently not considered so far in the distance distributivity literature, where the sorting key is syntactically embedded in the distributive share. Worked-out examples are provided with Glue Semantics proofs.

Keywords: Distance Distributivity, Polish, Glue Semantics, LFG

1 Introduction

The aim of this paper is to provide a semantic analysis of basic distance distributivity facts in Polish. The phenomenon at hand is illustrated by the following examples from English, German and Polish; their common feature is that the distributive element (*each*, *jeweils*, *po*) combines directly with the distributed NP[1] (cf. *two sausages* in (1)) and that the plural NP denoting the restriction of the distribution (cf. *boys* in (1)) may be expressed at some distance from the distributive element.

(1) The boys have bought two sausages each.

(2) Die Jungen haben jeweils zwei Würstchen gekauft. (German; Zimmermann 2002:37)
the boys have DISTR two sausages bought
'The boys have bought two sausages each.'

(3) Chłopcy kupili po dwie kiełbaski. (Polish)
boys bought DISTR two sausages
'The boys (have) bought two sausages each.'

Following Choe 1987, Zimmermann 2002 and subsequent literature, the phrase denoting distributed objects (*two sausages* here) will be called *distributive* (or *distributed*) *share*, and the phrase denoting the set over which distribution takes place (*boys* above) will be called *sorting* (or *distributive*) *key*.

Zimmermann 2002 – couched in the transformational grammar and roughly following the approach to semantics outlined in Heim and Kratzer 1998 – remains the most comprehensive account of distance distributivity in German and cross-linguistically, but it's not without problems. Dotlačil 2012 notes that on Zimmermann's account the relation between the distributive share and the sorting key must be expressed by a constituent in the syntactic tree (e.g., such a constituent exists for *have bought* in (1)), but examples where no such constituent may be

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[1] Polish is a determinerless language, hence the use of NP rather than DP here.

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posited are easily found, as in *Alex and Sasha visited the capitals of three states each* (there is no constituent corresponding exactly to *visited the capitals of*, even at LF, as movement out of NPs is prohibited). Moreover, while Zimmermann (2002) seeks to provide an account not relying on LF movement, he acknowledges that his analysis must assume such covert movement for some occurrences of *jeweils* (see his § 2.4.2 in ch. V, pp. 271ff.), e.g., for the following example:

(4) Jeweils ein Offizier begleitete die Ballerinen nach Haus. (Zimmermann 2002:27)
DISTR one officer accompanied the ballerinas to home
'Each ballerina was accompanied home by one officer.'

Finally, his analysis does not handle inverse linking cases where the sorting key is syntactically embedded in the distributive share, as in the Polish example (5) (whose schematic syntactic structure is given in (6)) or the corresponding German example (7):

(5) Przybyło po 3 przedstawicieli 25 krajów. (Polish)
arrive.PAST DISTR 3 representatives 25.GEN countries.GEN
'3 representatives arrived from each of 25 countries.'

(6) Przybyło [po [3 [przedstawicieli [25 krajów]]]].

(7) Jeweils 3 Abgeordnete aus 25 Ländern trafen ein. (German; Malte Zimmermann, p.c.)
DISTR 3 representatives from 25 countries arrived

To the best of our knowledge such constructions – and the difficulties they cause – have not been noticed in the distance distributivity literature so far.

We propose an analysis which is free from such problems: it does not assume that the relation between the distributive share and the sorting key is expressed by a syntactic constituent, it is uniformly expressed at the interface between the level of grammatical functions and the semantic level, and it correctly handles constructions exemplified by (5)–(7).

The main idea of the account is that the semantic impact of *po* activates only once the distributive share combines semantically with the verb and creates a property S, e.g., once the meaning of *Przybyło 3 przedstawicieli* '$\lambda Y.$ 3 representatives of Y arrived' in (5) above becomes available, but before the meaning of the sorting key, *25 krajów* '25 countries', is consumed. The meaning of *po* combines with this property S, holding of some Y, and produces a new property, which is just like S but holds of all singleton subsets of Y. This new property combines with the sorting key, giving the appropriate meaning; in this case, the meaning that 'for each of 25 countries, 3 representatives arrived'.

The remainder of this paper is structured as follows. Polish distance distributivity facts are outlined in § 2. A brief introduction to Glue Semantics follows in § 3. The analysis, together with some worked-out examples (including (5) above), is presented in § 4. Finally, § 5 concludes the paper.

2 Distance distributivity in Polish

The syntactic behaviour of the distributive **po** in Polish is complex.[2] Przepiórkowski 2013 shows that at least three morphosyntactically different distributive lexemes **po** exist in Polish, illustrated below.[3]

[2]Lemmata denoting lexemes are written in **bold** here, and particular word forms are written in *italics*.

[3]The first two examples, (8)–(9), are constructed but uncontroversial. As mentioned below, the acceptability status of examples such as (10) is disputed, so this example is attested; NKJP stands for *Narodowy Korpus Języka*

(8) Dałem im po jabłku.
gave-I them.DAT DISTR apple.LOC
'I gave them an apple each.'

(9) Dałem im po dwa jabłka.
gave-I them.DAT DISTR two.ACC apples.ACC
'I gave them two apples each.'

(10) …nagroda należy się po trzem osobom z każdej klasy…
reward is due to DISTR three.DAT person.DAT.PL from each class
'Three people from each class deserve a reward.' (NKJP)

When *po* combines with a non-numeral nominal phrase, as in (8), this phrase must occur in the locative case, which in Polish is reserved for complements of some prepositions. Such *po*+NP phrases are restricted to so-called structural case positions (nominative, accusative, genitive of negation). The situation is much more complex when the distributive *po* combines with a numeral phrase. In some positions *po* behaves like a preposition assigning the accusative case; this is illustrated in (9), where case would remain accusative even if the verb was negated, cf. (11a) below. This shows that the NumP *dwa jabłka* 'two apples' receives its case from *po*, as otherwise it would bear the genitive of negation, as in (11b).

(11) a. Nie dałem im po dwa/*dwóch jabłka/*jabłek.
NEG gave-I them.DAT DISTR two.ACC/*GEN apples.ACC/*GEN
'I didn't give them two apples (each).'

b. Nie dałem im dwóch/*dwa jabłek/*jabłka.
NEG gave-I them.DAT two.GEN/*ACC apples.GEN/*ACC
'I didn't give them two apples.'

On the other hand, (10) illustrates that *po* sometimes does not assign case and may be transparent to case assignment; the dative on *trzem osobom* 'three people' is assigned by the verb. While similar examples may also be found for other morphological cases, including instrumental, genitive and locative, they are often judged marginal or downright unacceptable, which shows that the availability of this third lexeme **po** is restricted.

Despite such morphosyntactic idiosyncrasies, Przepiórkowski 2013 in the HPSG settings and Przepiórkowski and Patejuk 2013 within LFG, provide a unified analysis of the three lexemes **po** which treats all of them as heads. Hence, in the remainder of this paper we will not distinguish them and assume that the phrase *po* combines with its object.

Polish patterns with German rather than English in allowing the distributive share in the subject position. The classical – in Polish linguistics – paper Łojasiewicz 1979 cites, *inter alia*, the following examples with post-verbal subjects:[4]

Polskiego 'National Corpus of Polish' (http://nkjp.pl/; Przepiórkowski et al. 2012). Henceforth, Polish examples will not be explicitly marked as such.

[4]The phrase *dwa fotele* 'two armchairs' in (13) is not marked for case, as it is not clear whether this phrase occurs in the nominative or the accusative here; Przepiórkowski 2013 and Przepiórkowski and Patejuk 2013 argue for the accusative, despite appearances to the contrary.

(12) Z drzew spadło po jabłku.
from trees fell DISTR apple.LOC
'An apple fell from each tree.'

(13) W pokojach będą po dwa fotele.
in rooms be.FUT DISTR two armchairs
'There will be two armchairs in each room.'

Such cases pose no problem for the analysis proposed below.

One aspect of distance distributivity in Polish that is not considered here is the possibility of distribution over events. The argument that distributive elements like the German **jeweils** may quantify over events comes from examples such as (14) adduced by Moltmann 1997 (here cited after Zimmermann 2002:28):

(14) Peter hat Maria aus jeweils zwei Gründen kritisiert und gelobt. (German)
Peter has Maria for DISTR two reasons criticised and praised
'Peter has criticised and praised Maria for two reasons respectively.'

This sentence means that for each of the two events involving Peter as an agent and Maria as a patient, namely, that of criticising and that of praising, Peter had two reasons to be so involved in them. Similarly, the only way to interpret Zimmermann's (2002) example (15) is to assume a contextually given set of events of the Pope's travels that **jeweils** quantifies over.

(15) Der Papst ist in jeweils drei Länder gefahren. (German)
the Pope has to DISTR three countries travelled
'The Pope has travelled to three countries each.'

Similar examples can be constructed in Polish:

(16) Piotr miał po dwa powody by chwalić i krytykować Marię.
Piotr had DISTR two reasons to praise and criticise Maria.
'Peter had two reasons each to criticise and to praise Maria.'

(17) Papież zwiedzał po trzy kraje.
Pope visited DISTR three countries
'The Pope visited three countries each time.'

Nevertheless, we assume simplistic eventless representations here and do not treat such cases of distributivity over events.

3 Glue Semantics

In traditional approaches to compositionality, e.g., Heim and Kratzer 1998, meanings combine when they are expressed by siblings in a constituency tree. By contrast, in Glue Semantics (Dalrymple 1999, 2001) coupled with Lexical-Functional Grammar (Bresnan 2001, Dalrymple 2001), meanings correspond to f(unctional)-structures, rather than to c(onstituent)-structures, and meaning representations are paired with glue formulae specifying how these meanings combine with which other meanings. Any pair consisting of a meaning representation and a glue formula is called a *meaning constructor*.

For example, the glue part of the meaning constructor of various forms of **yawn** is:

(18) $(\uparrow \textsc{subj})_\sigma \multimap \uparrow_\sigma$

As usual in LFG, the up arrow ↑ in a lexical entry denotes the f-structure of the word, (↑ SUBJ) denotes the f-structure of the subject of this word, and σ is a function from f-structures to s(emantic)-structures. In effect, (18) is saying that by consuming the s-structure corresponding to the subject of a form of **yawn** such as *yawned*, we may produce the s-structure corresponding to *yawned* and, hence, to the whole clause headed by *yawned* (in LFG heads normally share their f-structure with their projections).

This mode of composition remains true regardless of specific tree configurations. For example, when *yawn* is a complement of a control verb, its covert subject is never realised in the c(onstituent)-structure, according to standard LFG analyses, but it is still present in its f-structure, as the value of the SUBJ attribute, so (18) is still relevant.

Glue Semantics is resource-sensitive: once a semantic resource is consumed, it cannot be reused. Dually, all semantic resources introduced by lexical items (or otherwise; semantic resources may be introduced constructionally) must be consumed in a derivation of the semantic resource of the whole sentence. For example, assuming that *David* introduces a single semantic resource of the right type, this resource is consumed by the resource of *yawned* given in (18), producing the resource $\uparrow_\sigma$ for the sentence *David yawned*; as this is the only resource left, the derivation succeeds.

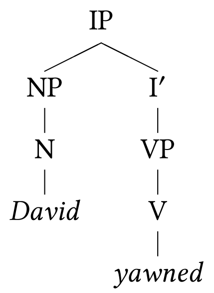
The other part of the meaning constructor is a formula in any language that allows application and abstraction, e.g., the language of the first-order predicate logic with lambda calculus. For example, the meaning of *David* can be defined as a logical constant, *David*, and the meaning of *yawned* can be defined as usual, as $\lambda X.yawn(X)$ (ignoring event variables, semantic roles, tense and aspect, etc.). In complete meaning constructors, the meaning part is separated from the glue part by the uninterpretable colon character, :, so the complete meaning constructors for *David* and *yawned* are as in the second lines of the following lexical entries:

(19) *David* N (↑ PRED) = 'DAVID'
$David : \uparrow_\sigma$

(20) *yawned* V (↑ PRED) = 'YAWN<SUBJ>'
$\lambda X.yawn(X) : (\uparrow \textsc{subj})_\sigma \multimap \uparrow_\sigma$

According to these lexical entries and standard LFG constituency rules, *David yawned* receives the c-structure displayed in (21) and the f-structure in (22); moreover, given this f-structure, meaning constructors are instantiated as in (23):[5]

(21)



(22) $\boxed{0}\begin{bmatrix} \textsc{pred} & \text{‘}\textsc{yawn}\langle\boxed{1}\rangle\text{’} \\ \textsc{subj} & \boxed{1}\begin{bmatrix} \textsc{pred} & \text{‘}\textsc{David}\text{’} \end{bmatrix} \end{bmatrix}$

(23) **[David]** $David : \boxed{1}_\sigma$
[yawned] $\lambda X.yawn(X) : \boxed{1}_\sigma \multimap \boxed{0}_\sigma$

[5] We adopt here the HPSG convention of naming feature structures with boxed numbers and of signalling structure-sharing by the repeated occurrence of a boxed number (cf. $\boxed{1}$ in (22)). Labels of meaning constructors are written in **[bold-in-square-brackets]**.

Now, using one of the proof rules of Glue Semantics, namely, the Implication Elimination rule in (24), and performing the usual β-reduction, the meaning of *David yawned* may be derived from the meaning constructors in (23) as shown in (25):

(24) $$\frac{a : A \qquad f : A \multimap B}{f(a) : B} \multimap_{\mathcal{E}}$$

(25) $$\frac{David : \boxed{1}_\sigma \qquad \lambda X.yawn(X) : \boxed{1}_\sigma \multimap \boxed{0}_\sigma}{yawn(David) : \boxed{0}_\sigma} \multimap_{\mathcal{E}}$$

Since both meaning resources introduced by lexical items, $\boxed{1}_\sigma$ and $\boxed{1}_\sigma \multimap \boxed{0}_\sigma$, were consumed in this proof, and the only meaning resource produced, $\boxed{0}_\sigma$, corresponds to the f-structure of the whole sentence, this is a valid proof that the meaning side of the whole sentence is *yawn*(*David*).

Obviously, we cannot do justice to Glue Semantics within the confines of this paper; the above is only meant to make the analysis below more accessible to motivated readers not familiar with this approach. The best introduction to Glue Semantics may still be found in the classical LFG textbook of Dalrymple 2001, on which the above exposition is based, but see also Andrews 2010 and Asudeh 2011 (ch. 4). Early influential papers are gathered in Dalrymple 1999, but they may be a little hard for an uninitiated reader, as they use a different – perhaps less transparent – notation; the exception is Dalrymple et al. 1999a, which introduces the notation adopted in subsequent work on Glue Semantics (including the current paper).

The glue side of meaning constructors is a fragment of linear logic (Girard 1987). Resources are understood here as LFG semantic structures projected from functional structures, but that does not mean that Glue Semantics is necessarily tightly coupled with LFG; versions of this approach have been proposed for other grammatical formalisms, including Head-driven Phrase Structure Grammar (Asudeh and Crouch 2002) and Lexicalized Tree Adjoining Grammar (Frank and van Genabith 2001). Also, while the meaning side adopted here is a version of the language of predicate logic with lambdas, this is not a necessity. Instead, Intensional Logic is employed in Dalrymple et al. 1999c and various derivatives of Discourse Representation Theory are used in Dalrymple et al. 1999b, Crouch and van Genabith 1999 and – more recently – in Haug 2013.

4 Analysis

4.1 Preliminaries

Let us first consider the two run-of-the-mill examples below:

(26) Chłopcy mają po dwa tatuaże.
boys.NOM have.PL DISTR two.ACC tattoos.ACC
'(The/Some) boys have two tattoos each.'

(27) Piotr kupił dziewczynom po róży.
Piotr.NOM bought.SG girls.DAT DISTR rose.LOC
'Peter bought (the/some) girls a rose each.'

In both examples the *po*-phrase (the distributive share) occupies the position of the direct object of the verb; the purely morphosyntactic – and inconsequential for the analysis below – difference between the accusative case of *dwa tatuaże* 'two tattoos' in (26) and the locative case of *róży* 'rose' in (27) is explained in § 2. The sorting key is expressed by the subject *Chłopcy* 'boys' in (26) and by the indirect object *dziewczynom* 'girls' in (27).

The intended meaning representations of these two examples are given below:

(28) intended meaning representation of (26):
$exists(Z, boy^s(Z) \wedge |Z| > 1,$
$all(X, |X| = 1 \wedge X \subset Z,$
$exists(V, |V| = 2 \wedge tattoo^s(V), have(X, V))))$

(29) intended meaning representation of (27):
$exists(Z, girl^s(Z) \wedge |Z| > 1,$
$all(X, |X| = 1 \wedge X \subset Z,$
$exists(V, |V| = 1 \wedge rose^s(V), bought(p, V, X))))$

In fact, both examples taken out of context are similarly ambiguous: the plural bare NPs (*Chłopcy* 'boys' and *dziewczynom* 'girls') may be interpreted either as indefinites or as definites. For reasons of simplicity, both indefinites and definites are represented as generalised quantifiers in the current paper; the former are approximated by the existential quantifer *exists*, as in the representations above, and the latter will be represented below via the *iota* relation.

As common in LFG and Glue Semantics, generalised quantifiers are represented here as *pair quantifiers*, i.e., as relations between an individual and two propositions involving that individual, so that *Everyone yawned* has the representation $all(X, person(X), yawn(X))$ (Dalrymple 2001:227). Moreover, we follow Dotlačil 2012 and earlier work on treating type *e* objects as sets, and properties – as sets of such sets. For example, boy^s is the property of being a non-empty set of boys – either a singleton or a set of higher cardinality (the superscript *s* indicates the possible plural) – and $\lambda Z. |Z| > 1 \wedge boy^s(Z)$ is the property of being a set of at least two boys. On this view, the standard inclusion relation $\subseteq$ is defined on type *e* objects.

How do these meaning representations differ from meanings of corresponding examples without the distributive element? The relevant examples and intended meanings (assuming the existential closure of all bare NPs) are given below.

(30) a. Chłopcy mają dwa tatuaże. (cf. (26))
boys.NOM have.PL two.ACC tattoos.ACC
'(The/Some) boys have two tattoos.'

b. $exists(Z, boy^s(Z) \wedge |Z| > 1,$ (cf. (28))
$exists(V, |V| = 2 \wedge tattoo^s(V), have(Z, V))))$

(31) a. Piotr kupił dziewczynom różę. (cf. (27))
Piotr.NOM bought.SG girls.DAT rose.ACC
'Peter bought a rose for (the/some) girls.'

b. $exists(Z, girl^s(Z) \wedge |Z| > 1,$ (cf. (29))
$exists(V, |V| = 1 \wedge rose^s(V), bought(p, V, Z))))$

The difference between the meaning representations in (30b)–(31b) above and the earlier representations in (28)–(29) should make the impact of the distributive **po** clear: it takes a property holding of some set and transforms it into an analogous property holding of each singleton subset of the set. We formalise this observation in the following subsection.

4.2 Semantics of **po** and worked-out example

The first version of the meaning constructor for **po**, labelled as **[distr]**, is given below:[6]

(32) **[distr]** $\lambda S.\lambda Z.all(X, |X| = 1 \wedge X \subset Z, S(X)) : \forall G,H.\ [G \multimap H] \multimap [G \multimap H]$

The meaning part (on the left of the colon) directly reflects the considerations of the previous subsection: **po** takes a property S and returns a property that holds of Z if and only if S holds of all singleton (proper[7]) subsets of Z. The glue part (on the right of the colon) says that **po** is an identity function on semantic resources corresponding to properties: it consumes a resource $[G \multimap H]$ (for any G and H of appropriate types) in order to produce the same resource. Hence, **po** as construed above may combine with just any property in the sentence; as we will see below, this analysis is too permissive and will be further constrained in § 4.4.

We will illustrate the analysis in detail on the basis of example (26), repeated below (with the additional assumption that the subject is to be understood existentially):

(26′) Chłopcy mają po dwa tatuaże.
boys.NOM have.PL DISTR two.ACC tattoos.ACC
'Some boys have two tattoos each.'

As usual in LFG and Glue Semantics, the two common nouns occurring in this sentence have the following lexical entries (ignoring morphosyntactic features such as case or gender):

(33) *chłopcy* N (↑ PRED) = 'BOYS'
$\lambda X.boy^s(X) \wedge |X| > 1 : (\uparrow_\sigma \text{ VAR}) \multimap (\uparrow_\sigma \text{ RESTR})$

(34) *tatuaże* N (↑ PRED) = 'TATTOOS'
$\lambda X.tattoo^s(X) \wedge |X| > 1 : (\uparrow_\sigma \text{ VAR}) \multimap (\uparrow_\sigma \text{ RESTR})$

The glue sides show that semantic structures may have some internal structure – s-structures of common nouns have the attributes VAR and RESTR, representing a variable (of type e) and a restriction (of type t).

Simplifying somewhat, we treat cardinals as existential quantifiers:

(35) *dwa* Num (↑ SPEC) = 2
$\lambda R.\lambda S.exists(Y, |Y| = 2 \wedge R(Y), S(Y)) :$
$[(\uparrow_\sigma \text{ VAR}) \multimap (\uparrow_\sigma \text{ RESTR})] \multimap [\forall H.[\uparrow_\sigma \multimap H] \multimap H]$

While there are syntactic arguments that numerals take the following NPs as complements, i.e., that phrases of the form "Num+NP" are really numeral phrases (Saloni and Świdziński 2001, Przepiórkowski 1999), we simplify here by treating the numeral and the following noun as co-heads. Given the c-structure rule in (36), we get the f-structure for *dwa tatuaże* 'two tattoos' shown in (37):

(36) NumP → Num N
↑=↓ ↑=↓

(37) $\boxed{3}$ [SPEC '2' ; PRED 'TATTOOS']

[6]The meaning side is essentially the semantic representation of the abstract DIST(ributivity) operator proposed by Link 1991. The arguments given by Zimmermann 2002:68–69 that the German **jeweils** is not an overt realisation of DIST do not bear on the choice of this meaning representation here.

[7]A more complete representation would also include a presupposition that Z is indeed a set of more than one element.

Given this f-structure, all occurrences of ↑ in (34) and in (35) instantiate to $\boxed{3}$, so we can construct the following proof for the meaning of *dwa tatuaże* 'two tattoos':[8]

(38)
$$\dfrac{\begin{array}{l}\lambda R.\lambda S.exists(Y, |Y| = 2 \wedge R(Y), S(Y)) :\\ [(\boxed{3}_\sigma \textsc{var}) \multimap (\boxed{3}_\sigma \textsc{restr})] \multimap [\forall H.[\boxed{3}_\sigma \multimap H] \multimap H]\end{array} \qquad \begin{array}{l}\lambda X.tattoo^s(X) \wedge |X| > 1 :\\ (\boxed{3}_\sigma \textsc{var}) \multimap (\boxed{3}_\sigma \textsc{restr})\end{array}}{\begin{array}{l}\lambda S.exists(Y, |Y| = 2 \wedge tattoo^s(Y), S(Y)) :\\ \forall H.[\boxed{3}_\sigma \multimap H] \multimap H\end{array}} \multimap_{\mathcal{E}}$$

The only missing lexical entries needed to analyse (26) are that of the main verb, *mają* 'have', cf. (39), and that of *po*, cf. (40):

(39) *mają* V $(\uparrow \textsc{pred}) = \text{`}\textsc{have<subj,obj>}\text{'}$
$\lambda X.\lambda Y.have(X, Y) : (\uparrow \textsc{subj})_\sigma \multimap [(\uparrow \textsc{obj})_\sigma \multimap \uparrow_\sigma]$

(40) *po* P $(\uparrow \textsc{pred}) = \text{`}\textsc{po<obj>}\text{'}$
$(\uparrow \textsc{obj})_\sigma = \uparrow_\sigma$
$\lambda S.\lambda Z.all(X, |X| = 1 \wedge X \subset Z, S(X)) : \forall G, H.\ [G \multimap H] \multimap [G \multimap H]$

The lexical entry of the verb should be self-explanatory at this stage: the semantic resources of the subject and the object must be consumed to produce a semantic resource corresponding to the verb (and, hence, to the whole sentence headed by this verb). On the other hand, *po* is analysed as a preposition here,[9] but the only semantic resource it introduces is the general $\forall G, H.\ [G \multimap H] \multimap [G \multimap H]$. In particular, it does not consume the semantic resource of its object, but rather equates its own s-structure with that of this object, in effect sharing with the object a single semantic resource.

These lexical entries, together with standard c-structure rules, produce the following f-structure for the complete sentence (26):

(41)
$$\boxed{0}\left[\begin{array}{ll}\textsc{pred} & \text{`}\textsc{have}\langle\boxed{1},\boxed{2}\rangle\text{'}\\ \textsc{subj} & \boxed{1}\left[\begin{array}{ll}\textsc{pred} & \text{`}\textsc{boys}\text{'}\end{array}\right]\\ \textsc{obj} & \boxed{2}\left[\begin{array}{ll}\textsc{pred} & \text{`}\textsc{po}\langle\boxed{3}\rangle\text{'}\\ \textsc{obj} & \boxed{3}\left[\begin{array}{ll}\textsc{spec} & \text{`2'}\\ \textsc{pred} & \text{`}\textsc{tattoos}\text{'}\end{array}\right]\end{array}\right]\end{array}\right]$$

The constraint $(\uparrow \textsc{obj})_\sigma = \uparrow_\sigma$ instantiates thus to $\boxed{3}_\sigma = \boxed{2}_\sigma$, so the conclusion of the subproof (38) is equivalent to (42). This conclusion may be combined with the meaning of *mają* 'have', instantiated to (43), rendering the meaning of *mają dwa tatuaże* 'have two tattoos' in (44):

(42) **[two-tattoos]** $\lambda S.exists(Y, |Y| = 2 \wedge tattoo^s(Y), S(Y)) : \forall H.[\boxed{2}_\sigma \multimap H] \multimap H$

(43) **[have]** $\lambda X.\lambda Y.have(X, Y) : \boxed{1}_\sigma \multimap [\boxed{2}_\sigma \multimap \boxed{0}_\sigma]$

(44) **[have-two-tattoos]** $\lambda X.exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X, Y)) : \boxed{1}_\sigma \multimap \boxed{0}_\sigma$

[8] Each meaning constructor is broken into two lines for typographical reasons. We also drop the conjunct $|Y| > 1$ in the conclusion, as it follows from $|Y| = 2$.

[9] As discussed in § 2, there are three different lexemes **po** in Polish, but they are all analysed as heads, so the lexical entry in (40) is a sufficiently good approximation of all of them.

A proof of (44) involves another Glue Semantics proof rule, Implication Introduction, which says that if the introduction of an assumption $[x : A]$ leads to a proof of $f : B$ then $\lambda x.f : A \multimap B$ is proved:

(45)
$$\frac{\begin{array}{c}[x : A]^1 \\ \vdots \\ f : B\end{array}}{\lambda x.f : A \multimap B} \multimap_{\mathcal{I},1}$$

Given this proof rule, the proof of (44) proceeds as follows:

(46)
$$\cfrac{\cfrac{[X : \boxed{1}_\sigma]^1 \quad \begin{array}{l}\lambda X.\lambda Y.have(X,Y) : \\ \boxed{1}_\sigma \multimap [\boxed{2}_\sigma \multimap \boxed{0}_\sigma]\end{array}}{\lambda Y.have(X,Y) : \boxed{2}_\sigma \multimap \boxed{0}_\sigma} \multimap_{\mathcal{E}} \quad \begin{array}{l}\lambda S.exists(Y, |Y| = 2 \wedge tattoo^s(Y), S(Y)) : \\ \forall H.[\boxed{2}_\sigma \multimap H] \multimap H\end{array}}{\cfrac{exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X,Y)) : \boxed{0}_\sigma}{\lambda X.exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X,Y)) : \boxed{1}_\sigma \multimap \boxed{0}_\sigma} \multimap_{\mathcal{I},1}} \multimap_{\mathcal{E}}$$

The conclusion of proof (46), i.e., the meaning constructor **[have-two-tatoos]** of (44), is of the form that may be combined with the meaning constructor for *po* given in (32) (and repeated in (40)):

(47)
$$\cfrac{\begin{array}{l}\lambda X.exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X,Y)): \\ \boxed{1}_\sigma \multimap \boxed{0}_\sigma\end{array} \quad \begin{array}{l}\lambda S.\lambda Z.all(X, |X| = 1 \wedge X \subset Z, S(X)): \\ \forall G,H.\ [G \multimap H] \multimap [G \multimap H]\end{array}}{\begin{array}{l}\lambda Z.all(X, |X| = 1 \wedge X \subset Z, exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X,Y))) : \\ \boxed{1}_\sigma \multimap \boxed{0}_\sigma\end{array}} \multimap_{\mathcal{E}}$$

Now we face an apparent problem, as – apart from the resource in the conclusion of proof (47) – the only other resource left is that of *chłopcy* 'boys', instantiated here to (48), and these two resources are incompatible (cannot be combined).

(48) **[boys]** $\lambda X.boy^s(X) \wedge |X| > 1 : (\boxed{1}_\sigma \text{VAR}) \multimap (\boxed{1}_\sigma \text{RESTR})$

However, as noted above, such bare NPs are understood as either existentially closed or as definites, so the grammar must provide appropriate meaning constructors transforming meanings of bare NPs into generalised quantifiers. In the case at hand, the meaning constructor that is needed is (compare this to the meaning of *dwa* 'two' in (35)):

(49) **[existential]** $\lambda R.\lambda S.exists(Z, R(Z), S(Z)) :$
$[(\boxed{1}_\sigma \text{VAR}) \multimap (\boxed{1}_\sigma \text{RESTR})] \multimap [\forall H.[\boxed{1}_\sigma \multimap H] \multimap H]$

Once this constructor is available, the existential meaning of *chłopcy* 'boys' may be derived using the Implication Elimination proof rule:

(50)
$$\cfrac{\begin{array}{l}\lambda X.boy^s(X) \wedge |X| > 1 : \\ (\boxed{1}_\sigma \text{VAR}) \multimap (\boxed{1}_\sigma \text{RESTR})\end{array} \quad \begin{array}{l}\lambda R.\lambda S.exists(Z, R(Z), S(Z)) : \\ [(\boxed{1}_\sigma \text{VAR}) \multimap (\boxed{1}_\sigma \text{RESTR})] \multimap [\forall H.[\boxed{1}_\sigma \multimap H] \multimap H]\end{array}}{\begin{array}{l}\lambda S.exists(Z, boy^s(Z) \wedge |Z| > 1, S(Z)) : \\ \forall H.[\boxed{1}_\sigma \multimap H] \multimap H\end{array}} \multimap_{\mathcal{E}}$$

Applying the same proof rule to the conclusions of (47) and (50), we obtain the same (up to variable names) meaning side as the intended meaning representation of (26), given in (28):

(51)
$$\dfrac{\begin{array}{l}\lambda Z.\mathit{all}(X, |X| = 1 \wedge X \subset Z,\\ \mathit{exists}(Y, |Y| = 2 \wedge \mathit{tattoo}^s(Y), \mathit{have}(X, Y))) :\\ \boxed{1}_\sigma \multimap \boxed{0}_\sigma\end{array} \quad \begin{array}{l}\lambda S.\mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, S(Z)) :\\ \forall H.[\boxed{1}_\sigma \multimap H] \multimap H\end{array}}{\begin{array}{l}\mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1,\\ \quad \mathit{all}(X, |X| = 1 \wedge X \subset Z,\\ \qquad \mathit{exists}(Y, |Y| = 2 \wedge \mathit{tattoo}^s(Y), \mathit{have}(X, Y)))) : \boxed{0}_\sigma\end{array}} \multimap_{\mathcal{E}}$$

The schematic structure of the whole proof is given below, with references to particular subproofs:

(52)
$$\dfrac{\dfrac{\dfrac{\textbf{[have]} \quad \dfrac{\textbf{[two]}\ \textbf{[tattoos]}}{\textbf{[two-tattoos]}}(38)}{\textbf{[have-two-tattoos]}}(46) \quad \textbf{[distr]}}{\textbf{[distr-have-two-tattoos]}}(47) \quad \dfrac{\textbf{[boys]}\ \textbf{[existential]}}{\textbf{[boys-existential]}}(50)}{\textbf{[boys-existential-distr-have-two-tattoos]}}(51)$$

Note that all resources introduced by lexical items have been consumed in the process and that the only resource left is $\boxed{0}_\sigma$, which corresponds to the complete sentence; hence, this is a linguistically valid proof (Asudeh 2011, ch. 5).

An analogous proof could be constructed for the definite reading of *chłopcy* 'boys', using the following meaning constructor instead of **[existential]** of (49):

(53) **[definite]** $\lambda R.\lambda S.\mathit{iota}(Z, R(Z), S(Z))$:
$[(\boxed{1}_\sigma \text{VAR}) \multimap (\boxed{1}_\sigma \text{RESTR})] \multimap [\forall H.[\boxed{1}_\sigma \multimap H] \multimap H]$

Such meaning constructors must be optionally available for any common noun. If the noun contributes to the restriction of a lexical quantifier, as in case of *tatuaże* 'tattoos' restricting the quantifier *dwa* 'two', such optional meaning constructors cannot be used – the lexical quantifier consumes the resources necessary to activate such meaning constructors. On the other hand, when there is no appropriate lexical quantifier, either the existential closure or the definiteness meaning constructor may activate and combine with the bare noun.[10]

4.3 Sorting key within distributive share

Let us now turn to (5), repeated below as (5′), where the sorting key, *25 krajów* '25 countries', is syntactically embedded within the phrase expressing the distributive share, *po 3 przedstawicieli 25 krajów* '3 representatives of (each of) 25 countries'; the schematic constituent structure is repeated as (6′).

(5′) Przybyło po 3 przedstawicieli 25 krajów. (Polish)
arrive.PAST DISTR 3 representatives 25.GEN countries.GEN
'3 representatives arrived from each of 25 countries.'

[10] We assume that such optional meaning constructors are introduced in lexical entries of common nouns, as part of a common noun template, so as to avoid missing generalisations (Dalrymple et al. 2004, Asudeh et al. 2013); another option would be to add them to appropriate c-structure rules.

(6′) Przybyło [po [3 [przedstawicieli [25 krajów]]]].

Lexical entries for *3* and *25* given below parallel that for *dwa* ‘two’ given in (35):

(54) *3* Num $(\uparrow \text{SPEC}) = 3$
$\lambda R.\lambda S.exists(Y, |Y| = 3 \wedge R(Y), S(Y))$:
$[(\uparrow_\sigma \text{VAR}) \multimap (\uparrow_\sigma \text{RESTR})] \multimap [\forall H.[\uparrow_\sigma \multimap H] \multimap H]$

(55) *25* Num $(\uparrow \text{SPEC}) = 25$
$\lambda R.\lambda S.exists(Y, |Y| = 25 \wedge R(Y), S(Y))$:
$[(\uparrow_\sigma \text{VAR}) \multimap (\uparrow_\sigma \text{RESTR})] \multimap [\forall H.[\uparrow_\sigma \multimap H] \multimap H]$

Similarly, the lexical entry for *krajów* ‘countries’ is analogous to those for *chłopcy* ‘boys’ and *tatuaże* ‘tattoos’ in (33)–(34), and the entry for *przybyło* ‘arrived’ is simpler than that for *mają* ‘have’ in (39), as it only takes one argument:

(56) *krajów* N $(\uparrow \text{PRED}) = \text{‘COUNTRIES’}$
$\lambda X.country^s(X) \wedge |X| > 1 : (\uparrow_\sigma \text{VAR}) \multimap (\uparrow_\sigma \text{RESTR})$

(57) *przybyło* V $(\uparrow \text{PRED}) = \text{‘ARRIVE<SUBJ>’}$
$\lambda X.arrive(X) : (\uparrow \text{SUBJ})_\sigma \multimap \uparrow_\sigma$

What is new in this example, is a relational noun, *przedstawicieli* ‘representatives’:[11]

(58) *przedstawicieli* N $(\uparrow \text{PRED}) = \text{‘REPRESENTATIVES<OBJ>’}$
$\lambda Y.\lambda X representative^s(X, Y) \wedge |X| > 1$:
$(\uparrow \text{OBJ})_\sigma \multimap [(\uparrow_\sigma \text{VAR}) \multimap (\uparrow_\sigma \text{RESTR})]$

The meaning constructor of (58) differs from that of (56) and other non-relational nouns in the additional requirement of the semantic resource corresponding to the argument of the relational noun.

With these lexical entries, as well as the lexical entry for *po* given in (40) above, the f-structure of (5) is as shown in (59).

(59)
$$\boxed{0}\left[\begin{array}{ll} \text{PRED} & \text{‘ARRIVED}\langle\boxed{1}\rangle\text{’} \\ \text{SUBJ} & \boxed{1}\left[\begin{array}{ll} \text{PRED} & \text{‘PO}\langle\boxed{2}\rangle\text{’} \\ \text{OBJ} & \boxed{2}\left[\begin{array}{ll} \text{SPEC} & \text{‘3’} \\ \text{PRED} & \text{‘REPRESENTATIVE}\langle\boxed{3}\rangle\text{’} \\ \text{OBJ} & \boxed{3}\left[\begin{array}{ll} \text{SPEC} & \text{‘25’} \\ \text{PRED} & \text{‘COUNTRY’} \end{array}\right] \end{array}\right] \end{array}\right] \end{array}\right]$$

The intended meaning of (5), given in (60), may be attained via the proof schematically shown in (61), where particular meaning constructors, as instantiated for (59), are given in (62)–(71).[12]

[11]We remain agnostic as to whether OBJ, assumed in (58), is really the right grammatical function for the complement of *przedstawicieli* ‘representatives’. Dalrymple et al. 1999c:57 and Dalrymple 2001:249 analyse arguments of English nouns *rumor* and *relative*, introduced by the prepositional markers *about* and *of*, as values of $\text{OBL}_{\text{ABOUT}}$ and OBL_{OF}, respectively.

[12]Again, we omit the condition $|X| > 1$ on plural nouns, once it follows from particular cardinalities contributed by the numerals.

(60) $exists(Z, |Z| = 25 \wedge country^s(Z),$
$\quad all(X, |X| = 1 \wedge X \subset Z,$
$\quad\quad exists(V, |V| = 3 \wedge representative^s(V, X), arrived(V)))) : \boxed{0}_\sigma$

(61)
$$\dfrac{\dfrac{\dfrac{\mathbf{[arrived]} \quad \dfrac{\mathbf{[3]}\ \mathbf{[representatives]}}{\mathbf{[3\text{-}representatives]}}\multimap_{\mathcal{E},\mathcal{E},\mathcal{I}}}{\mathbf{[arrived\text{-}3\text{-}representatives]}}\multimap_{\mathcal{E},\mathcal{E},\mathcal{I}} \quad \mathbf{[distr]}}{\mathbf{[distr\text{-}arrived\text{-}3\text{-}representatives]}}\multimap_{\mathcal{E}} \quad \dfrac{\mathbf{[25]}\ \mathbf{[countries]}}{\mathbf{[25\text{-}countries]}}\multimap_{\mathcal{E}}}{\mathbf{[25\text{-}countries\text{-}distr\text{-}arrived\text{-}3\text{-}representatives]}}\multimap_{\mathcal{E}}$$

(62) **[25]**
$\lambda R.\lambda S.exists(X, |X| = 25 \wedge R(X), S(X)) : [(\boxed{3}_\sigma \text{VAR}) \multimap (\boxed{3}_\sigma \text{RESTR})] \multimap [\forall H.[\boxed{3}_\sigma \multimap H] \multimap H]$

(63) **[countries]**
$\lambda X.country^s(X) \wedge |X| > 1 : (\boxed{3}_\sigma \text{VAR}) \multimap (\boxed{3}_\sigma \text{RESTR})$

(64) **[25-countries]**
$\lambda S.exists(X, |X| = 25 \wedge country^s(X), S(X)) : \forall H.[\boxed{3}_\sigma \multimap H] \multimap H$

(65) **[3]**
$\lambda R.\lambda S.exists(X, |X| = 3 \wedge R(X), S(X)) : [(\boxed{2}_\sigma \text{VAR}) \multimap (\boxed{2}_\sigma \text{RESTR})] \multimap [\forall H.[\boxed{2}_\sigma \multimap H] \multimap H]$

(66) **[representatives]**
$\lambda Y.\lambda X representative^s(X, Y) \wedge |X| > 1 : \boxed{3}_\sigma \multimap [(\boxed{2}_\sigma \text{VAR}) \multimap (\boxed{2}_\sigma \text{RESTR})]$

(67) **[3-representatives]** (note that $\boxed{1}_\sigma = \boxed{2}_\sigma$ by virtue of $(\uparrow \text{OBJ})_\sigma = \uparrow_\sigma$ in (40))
$\lambda Y.\lambda S.exists(X, |X| = 3 \wedge representative^s(X, Y), S(X)) : \forall H.\ \boxed{3}_\sigma \multimap [[\boxed{2}_\sigma \multimap H] \multimap H] \quad \equiv$
$\lambda Y.\lambda S.exists(X, |X| = 3 \wedge representative^s(X, Y), S(X)) : \forall H.\ \boxed{3}_\sigma \multimap [[\boxed{1}_\sigma \multimap H] \multimap H]$

(68) **[arrived]**
$\lambda X.arrived(X) : \boxed{1}_\sigma \multimap \boxed{0}_\sigma$

(69) **[arrived-3-representatives]**
$\lambda Y.exists(X, |X| = 3 \wedge representative^s(X, Y), arrived(X)) : \boxed{3}_\sigma \multimap \boxed{0}_\sigma$

(70) **[distr-arrived-3-representatives]** (see (32) above for **[distr]**)
$\lambda Z.all(X, |X| = 1 \wedge X \subset Z,$
$\quad exists(V, |V| = 3 \wedge representative^s(V, X), arrived(V))) : \boxed{3}_\sigma \multimap \boxed{0}_\sigma$

(71) **[25-countries-distr-arrived-3-representatives]**
$exists(Z, |Z| = 25 \wedge country^s(Z),$
$\quad all(X, |X| = 1 \wedge X \subset Z,$
$\quad\quad exists(V, |V| = 3 \wedge representative^s(V, X), arrived(V)))) : \boxed{0}_\sigma$

This is the only proof available for this sentence. This shows that the analysis proposed in the previous subsection correctly accounts for troublesome cases when the sorting key is embedded within the phrase expressing the distributive share.

4.4 *Constraining analysis*

While there is only one proof for the sentence considered in the previous subsection, § 4.3, the analysis overgenerates in many other cases. The problem is that the meaning of *po*, as given in (32) and (40), may combine with any (appropriately typed) property available in the sentence. In effect, sentence (26) considered in § 4.2 has another proof, leading to the incorrect meaning in (72), paraphrased as: *for either of some two tattoos, there are some boys that have it.*

(72) $$\begin{array}{l} \mathit{exists}(Y, |Y| = 2 \wedge \mathit{tattoo}^s(Y), \\ \quad \mathit{all}(X, |X| = 1 \wedge X \subset Y, \\ \quad\quad \mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,X)))) : \boxed{0}_\sigma \end{array}$$

The proof is analogous to (52), and it is given below, with references to subproofs:

(73) $$\dfrac{\dfrac{\dfrac{\textbf{[have]} \quad \dfrac{\textbf{[boys] [existential]}}{\textbf{[boys-existential]}}\,(50)}{\textbf{[boys-existential-have]}}\,(74) \quad \textbf{[distr]}}{\textbf{[distr-boys-existential-have]}}\,(75) \quad \dfrac{\textbf{[two] [tattoos]}}{\textbf{[two-tattoos]}}\,(38)}{\textbf{[two-tattoos-distr-boys-existential-have]}}\,(76)$$

(74) $$\dfrac{\dfrac{\dfrac{[Y : \boxed{2}_\sigma]^2 \quad \dfrac{[X : \boxed{1}_\sigma]^1 \quad \begin{array}{l}\lambda X.\lambda Y.\mathit{have}(X,Y) : \\ \boxed{1}_\sigma \multimap [\boxed{2}_\sigma \multimap \boxed{0}_\sigma]\end{array}}{\lambda Y.\mathit{have}(X,Y) : \boxed{2}_\sigma \multimap \boxed{0}_\sigma} \multimap_{\mathcal{E}}}{\dfrac{\mathit{have}(X,Y) : \boxed{0}_\sigma}{\lambda X.\mathit{have}(X,Y) : \boxed{1} \multimap \boxed{0}_\sigma} \multimap_{\mathcal{I},1}} \multimap_{\mathcal{E}} \quad \begin{array}{l}\lambda S.\mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, S(Z)) : \\ \forall H.[\boxed{1}_\sigma \multimap H] \multimap H\end{array}}{\dfrac{\mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,Y)) : \boxed{0}_\sigma}{\lambda Y.\mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,Y)) : \boxed{2}_\sigma \multimap \boxed{0}_\sigma} \multimap_{\mathcal{I},2}} \multimap_{\mathcal{E}}$$

(75) $$\dfrac{\begin{array}{l}\lambda Y.\mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,Y)) : \\ \boxed{2}_\sigma \multimap \boxed{0}_\sigma\end{array} \quad \begin{array}{l}\lambda S.\lambda Y.\mathit{all}(X, |X| = 1 \wedge X \subset Y, S(X)) : \\ \forall G,H.\ [G \multimap H] \multimap [G \multimap H]\end{array}}{\begin{array}{l}\lambda Y.\mathit{all}(X, |X| = 1 \wedge X \subset Y, \mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,X))) : \\ \boxed{2}_\sigma \multimap \boxed{0}_\sigma\end{array}} \multimap_{\mathcal{E}}$$

(76) $$\dfrac{\begin{array}{l}\lambda Y.\mathit{all}(X, |X| = 1 \wedge X \subset Y, \\ \mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,X))) : \\ \boxed{2}_\sigma \multimap \boxed{0}_\sigma\end{array} \quad \begin{array}{l}\lambda S.\mathit{exists}(Y, |Y| = 2 \wedge \mathit{tattoo}^s(Y), S(Y)) : \\ \forall H.[\boxed{2}_\sigma \multimap H] \multimap H\end{array}}{\begin{array}{l}\mathit{exists}(Y, |Y| = 2 \wedge \mathit{tattoo}^s(Y), \\ \quad \mathit{all}(X, |X| = 1 \wedge X \subset Y, \\ \quad\quad \mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,X)))) : \boxed{0}_\sigma\end{array}} \multimap_{\mathcal{E}}$$

A preliminary solution to this problem is inspired by the Glue Semantics approach to Negative Polarity Licensing proposed by Fry 1999. The intuition of this approach is that a Negative Polarity Item (NPI) “attaches” to its usual meaning a non-semantic resource which is transferred during the semantic derivation until it meets a licensor which discharges (i.e., consumes) it. In the case at hand, the distributive share acts as an NPI and the additional resource is discharged when the meaning of *po* combines with a meaning containing the contribution of this distributive share.

Technically, changes to the lexical entry of **po** are needed. First of all, another meaning constructor is added, called **[distr-q]** below, which transforms the quantifier over the object of **po** by attaching a non-semantic resource, call it l, to the semantic resource of the object, using the operator of multiplicative conjunction $\otimes$.[13,14] Hence, any meaning constructor of the form (77a) will be transformed into (77b).

(77) a. $\lambda R.\lambda S.Q(R,S) : [((\uparrow \text{OBJ})_\sigma \text{VAR}) \multimap ((\uparrow \text{OBJ})_\sigma \text{RESTR})] \multimap [\forall H.[(\uparrow \text{OBJ})_\sigma \multimap H] \multimap H]$

b. $\lambda R.\lambda S.Q(R,S) : [((\uparrow \text{OBJ})_\sigma \text{VAR}) \multimap ((\uparrow \text{OBJ})_\sigma \text{RESTR})] \multimap [\forall H.[(\uparrow \text{OBJ})_\sigma \otimes\ l \multimap H] \multimap H]$

Note that this meaning constructor is anchored in the meaning of the object of **po**, so it will only apply to quantifiers taking this object as their restriction, such as cardinal quantifiers in the examples above, the existential closure, etc.

Second, the previously employed meaning constructor **[distr]**, repeated in (78a), is modified as in (78b); this modified constructor will be called **[distr-l]** below:

(78) a. $\lambda S.\lambda Z.all(X, |X| = 1 \land X \subset Z, S(X)) : \forall G,H.\ [G \multimap H] \multimap [G \multimap H]$

b. $\lambda S.\lambda Z.all(X, |X| = 1 \land X \subset Z, S(X)) : \forall G,H.\ [G \multimap H \otimes l] \multimap [G \multimap H]$

The new correct proof tree for the sentence (26) considered in § 4.2 is given in (79), with most relevant conclusions of particular subproofs shown in (80)–(84).

(79)
$$\dfrac{\dfrac{\dfrac{\textbf{[have]} \quad \dfrac{\dfrac{\textbf{[two][distr-q]}}{\textbf{[two}'\textbf{]}}\multimap_{\mathcal{E}} \quad \textbf{[tattoos]}}{\textbf{[two}'\textbf{-tattoos]}}\multimap_{\mathcal{E}}}{\textbf{[have-two}'\textbf{-tattoos]}}\multimap_{\mathcal{E},\mathcal{E},\mathcal{I}} \quad \textbf{[distr-l]}}{\textbf{[distr-have-two-tattoos]}}\multimap_{\mathcal{E}} \quad \dfrac{\textbf{[boys] [existential]}}{\textbf{[boys-existential]}}(50)}{\textbf{[boys-existential-distr-have-two-tattoos]}}(51)$$

(80) **[two′]**
$\lambda R.\lambda S.exists(Y, |Y| = 2 \land R(Y), S(Y)) :$
$[(\boxed{3}_\sigma \text{VAR}) \multimap (\boxed{3}_\sigma \text{RESTR})] \multimap [\forall H.[\boxed{3}_\sigma \otimes l \multimap H] \multimap H]$

(81) **[two′-tattoos]**
$\lambda S.exists(Y, |Y| = 2 \land tattoo^s(Y), S(Y)) : \forall H.[\boxed{3}_\sigma \otimes l \multimap H] \multimap H \quad \equiv$
$\lambda S.exists(Y, |Y| = 2 \land tattoo^s(Y), S(Y)) : \forall H.[\boxed{2}_\sigma \otimes l \multimap H] \multimap H$

[13] Non-semantic atoms such as l are subformulae of the "old glue" language, in which other atomic formulae are pairs of (glue) types and meaning representations. In the "new glue" language used here, we put such non-atomic atoms in the glue part only; this is justified by the fact that they are preserved in (glue) type-projections but disappear in meaning-projections, as defined in § 4.3 of Dalrymple et al. 1999a. A disadvantage of this notational oversimplification is that the resulting meaning constructors look as if they were breaking the Curry-Howard correspondence. They do not; the use of multiplicative conjunction here is different from its use in the analysis of anaphora in Dalrymple 2001, where it corresponds to tuples on the meaning side.

[14] We assume that $\otimes$ binds stronger (has higher precedence) than $\multimap$.

(82) **[have]**
$\lambda X.\lambda Y.have(X,Y) : \boxed{1}_\sigma \multimap [\boxed{2}_\sigma \multimap \boxed{0}_\sigma]$ – from which follows (see (91) in the Appendix):
$\lambda X.\lambda Y.have(X,Y) : \boxed{1}_\sigma \multimap [\boxed{2}_\sigma \otimes l \multimap \boxed{0}_\sigma \otimes l]$

(83) **[have-two′-tattoos]**
$\lambda X.exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X,Y)) : \boxed{1}_\sigma \multimap \boxed{0}_\sigma \otimes l$

(84) **[distr-have-two-tattoos]** (see (78b) for **[distr-l]**)
$\lambda Z.all(X, |X| = 1 \wedge X \subset Z, exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X,Y))) : \boxed{1}_\sigma \multimap \boxed{0}_\sigma$

With the new meaning constructors for PO, the proof for (5) given in (61) in § 4.3 may be modified in a similar way. The key meaning constructors produced during such a proof are given below:

(85) **[3′]**
$\lambda R.\lambda S.exists(X, |X| = 3 \wedge R(X), S(X)) :$
$[(\boxed{2}_\sigma \text{VAR}) \multimap (\boxed{2}_\sigma \text{RESTR})] \multimap [\forall H.[\boxed{2}_\sigma \otimes l \multimap H] \multimap H]$

(86) **[3′-representatives]**
$\lambda Y.\lambda S.exists(X, |X| = 3 \wedge representative^s(X,Y), S(X)) : \forall H.\ \boxed{3}_\sigma \multimap [[\boxed{2}_\sigma \otimes l \multimap H] \multimap H] \quad \equiv$
$\lambda Y.\lambda S.exists(X, |X| = 3 \wedge representative^s(X,Y), S(X)) : \forall H.\ \boxed{3}_\sigma \multimap [[\boxed{1}_\sigma \otimes l \multimap H] \multimap H]$

(87) **[arrived]**
$\lambda X.arrived(X) : \boxed{1}_\sigma \multimap \boxed{0}_\sigma$ – from which follows (see the Appendix):
$\lambda X.arrived(X) : \boxed{1}_\sigma \otimes l \multimap \boxed{0}_\sigma \otimes l$

(88) **[arrived-3′-representatives]**
$\lambda Y.exists(X, |X| = 3 \wedge representative^s(X,Y), arrived(X)) : \boxed{3}_\sigma \multimap \boxed{0}_\sigma \otimes l$

(89) **[distr-arrived-3-representatives]** (see (78b) for **[distr-l]**)
$\lambda Z.all(X, |X| = 1 \wedge X \subset Z,$
$exists(V, |V| = 3 \wedge representative^s(V,X), arrived(V))) : \boxed{3}_\sigma \multimap \boxed{0}_\sigma$

On the other hand, the unwanted proof for the sentence (26), given in (73) above, is blocked now. The constructor **[distr-q]** may only combine with **[two]**, not with **[existential]**, so **[boys]**, **[existential]** and **[have]** must combine as in (73), resulting in the conclusion of (74), repeated in (90).

(90) $\lambda Y.exists(Z, boy^s(Z) \wedge |Z| > 1, have(Z,Y)) : \boxed{2}_\sigma \multimap \boxed{0}_\sigma$

This meaning constructor cannot be combined with **[distr-l]**, which now expects to consume a resource of the form $G \multimap H \otimes l$.

5 Conclusions

Analyses of distance distributivity, such as Choe 1987, Safir and Stowell 1988, Moltmann 1997, Zimmermann 2002 or Dotlačil 2012, have so far been formulated mainly within the transformational paradigm. In contrast, the current paper provides a non-transformational analysis, couched within Lexical Functional Grammar and coupled with the morphosyntactic account of Przepiórkowski and Patejuk 2013. On the semantic side, we employed the resource-sensitive approach of Glue Semantics. Empirically, the main point of this paper is the introduction – and successful analysis – of a construction troublesome for previous analyses, where the sorting key is syntactically embedded in the phrase expressing the distributive share.

The account proposed here is still at a relatively early stage of development. It remains to be seen whether the mechanism employed to harness overgeneration, introduced in § 4.4, is sufficiently general and robust. Moreover, we had nothing to say about distribution over events, witnessed in Polish and German, among other languages. Nevertheless, we hope that the current proposal provides a reasonable backbone to flesh out a more exhaustive constraint-based and resource-sensitive analysis of distance distributivity in Polish and other languages.

Appendix

A proof of the glue side of the inference employed in (82); see, e.g., Asudeh 2011:80–81 for the Conjunction Introduction $\otimes_{\mathcal{I}}$ and the Conjunction Elimination $\otimes_{\mathcal{E}}$ proof rules used here:

(91)
$$\dfrac{\dfrac{\dfrac{[\boxed{2}_\sigma \otimes l]^4 \qquad \dfrac{\dfrac{[\boxed{2}_\sigma]^2 \qquad \dfrac{[\boxed{1}_\sigma]^1 \quad \boxed{1}_\sigma \multimap [\boxed{2}_\sigma \multimap \boxed{0}_\sigma]}{\boxed{2}_\sigma \multimap \boxed{0}_\sigma}\multimap_{\mathcal{E}}}{\boxed{0}_\sigma}\multimap_{\mathcal{E}} \qquad [l]^3}{\boxed{0}_\sigma \otimes l}\otimes_{\mathcal{I}}}{\boxed{0}_\sigma \otimes l}\otimes_{\mathcal{E},2,3}}{\boxed{2}_\sigma \otimes l \multimap \boxed{0}_\sigma \otimes l}\multimap_{\mathcal{I},4}}{\boxed{1}_\sigma \multimap [\boxed{2}_\sigma \otimes l \multimap \boxed{0}_\sigma \otimes l]}\multimap_{\mathcal{I},1}$$

The meaning side is not affected by non-semantic atoms such as l here (Dalrymple et al. 1999a:277) and may be uniquely reconstructed from such a proof (Dalrymple et al. 1999a:271, Theorem 8).

The proof of the inference of $\boxed{1}_\sigma \otimes l \multimap \boxed{0}_\sigma \otimes l$ from $\boxed{1}_\sigma \multimap \boxed{0}_\sigma$ employed in (87) is analogous to the subproof of (91) which shows that $\boxed{2}_\sigma \otimes l \multimap \boxed{0}_\sigma \otimes l$ can be inferred from $\boxed{2}_\sigma \multimap \boxed{0}_\sigma$.

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