Distance Distributivity in Polish: Towards a Glue Semantics Approach

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We propose a novel syntactico-semantic analysis of distance distributivity in Polish and other languages, which is couched in Lexical Functional Grammar coupled with Glue Semantics. We introduce and analyse a troublesome construction, apparently not considered so far in the distance distributivity literature, where the sorting key is syntactically embedded in the distributive share. Worked-out examples are provided with Glue Semantics proofs.

Keywords: distance distributivity, Glue Semantics, LFG, Polish

1 Introduction

The aim of this paper is to provide a semantic analysis of some distance distributivity facts in Polish, including potentially problematic facts apparently not discussed previously either in the context of Polish or on the basis of other languages. Distance distributivity may be illustrated with the following examples from English, German, and Polish; their common feature is that the distributive element (*each*, *jeweils*, *po*) combines directly with the distributed NP[1] (e.g. *two sausages* in (1)) and that the plural NP denoting the restriction of the distribution (e.g. *boys* in (1)) may be expressed at some distance from the distributive element.

(1) The boys have bought two sausages each.

(2) Die Jungen haben jeweils zwei Würstchen gekauft. (German; Zimmermann 2002:37)
the boys have DISTR two sausages bought
'The boys have bought two sausages each.'

(3) Chłopcy kupili po dwie kiełbaski. (Polish)
boys bought DISTR two sausages
'The boys (have) bought two sausages each.'

Following Choe 1987, Zimmermann 2002 and subsequent literature, the phrase denoting the distributed objects (*two sausages* here) will be called the *distributive* (or *distributed*) *share*, and the phrase denoting the set over which distribution takes place (*boys* above) will be called the *sorting* (or *distributive*) *key*.

Zimmermann 2002 – couched in the transformational grammar and roughly following the approach to semantics outlined in Heim and Kratzer 1998 – remains the most comprehensive account of distance distributivity in German and cross-linguistically, but it is not without problems.[2] Dotlačil 2012 notes that on Zimmermann's account the relation between the distributive

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[1] Polish is a determinerless language, hence the use of NP rather than DP here.

[2] See Przepiórkowski 2014b for extended discussion.

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share and the sorting key must be expressed by a constituent in the syntactic tree (e.g. such a constituent exists for *have bought* in (1)), but examples where no such constituent may be posited are easily found, as in *Alex and Sasha visited the capitals of three states each* (there is no constituent corresponding exactly to *visited the capitals of*). Moreover, while Zimmermann (2002) seeks to provide an account not relying on LF movement, he acknowledges (sect. 2.4.2 of chap. V) that his analysis must assume such covert movement for some occurrences of *jeweils*, e.g. in (4) (Zimmermann 2002:269):

(4) Jeweils zwei Offiziere begleiteten die Ballerinen nach Haus. (German)
DISTR two officers accompanied the ballerinas to home
'Each ballerina was accompanied home by two officers.'

Finally, his analysis does not handle inverse linking cases where the sorting key is syntactically embedded in the distributive share, as in the Polish example (5) (whose schematic syntactic structure is given in (6)) or the corresponding German example (7) (Malte Zimmermann, p.c.):[3]

(5) Przybyło po 3 przedstawicieli 25 krajów. (Polish)
arrive.PAST DISTR 3 representatives 25.GEN countries.GEN
'3 representatives arrived from each of 25 countries.'

(6) Przybyło [po [3 [przedstawicieli [25 krajów]]]].

(7) Jeweils 3 Abgeordnete aus 25 Ländern trafen ein. (German)
DISTR 3 representatives from 25 countries arrived

To the best of our knowledge such constructions – and the difficulties they cause – have not been noticed in the distance distributivity literature so far.

We propose an analysis which is free from such problems: it does not assume that the relation between the distributive share and the sorting key is expressed by a syntactic constituent, it is uniformly formulated at the interface between the level of grammatical functions and the semantic level, and it correctly handles constructions exemplified by (5) and (7).

The main idea of the account is this: the semantic impact of *po* activates only once the distributive share combines semantically with the verb and creates a property. For example, in case of (5), the meaning of *Przybyło 3 przedstawicieli*, '$\lambda Y.$ 3 representatives of Y arrived', is derived first. Then, the meaning of *po* combines with this property, let us call it S, holding of some set Y, and produces a new property, which is just like S but holds of each element of Y: '$\lambda Y.$ for each element y of Y, 3 representatives of y arrived'. Finally, this new property combines with the sorting key *25 krajów* '25 countries', resulting in the meaning: 'for each of 25 countries, 3 representatives arrived'.

The remainder of this paper is structured as follows. Polish distance distributivity facts are outlined in section 2. A brief introduction to Glue Semantics follows in section 3. The analysis, together with some worked-out examples (including (5) above), is presented in section 4. Finally, section 5 concludes the paper.

[3] In order to increase clarity and shorten the textual form of the examples, numbers as written with digits here; the fully spelled-out form of (5) is: *Przybyło po trzech przedstawicieli dwudziestu pięciu krajów.*

2 Distance distributivity in Polish

The syntactic behaviour of the distributive *po* in Polish is complex. Przepiórkowski 2013 shows that at least three morphosyntactically different distributive lexemes *po* exist in Polish, illustrated below.[4]

(8) Dałem im po jabłku.
gave-I them.DAT DISTR apple.LOC
'I gave them an apple each.'

(9) Dałem im po dwa jabłka.
gave-I them.DAT DISTR two.ACC apples.ACC
'I gave them two apples each.'

(10) …nagroda należy się po trzem osobom z każdej klasy…
reward is due to DISTR three.DAT person.DAT.PL from each class
'Three people from each class deserve a reward.' (NKJP)

When *po* combines with a non-numeral nominal phrase, as in (8), this phrase must occur in the locative case, which in Polish is reserved for complements of some prepositions. Such *po*+NP phrases are restricted to so-called structural case positions (nominative, accusative, genitive of negation). The situation is much more complex when the distributive *po* combines with a numeral phrase. In some positions *po* behaves like a preposition assigning the accusative case; this is illustrated in (9), where case would remain accusative even if the verb was negated, cf. (11a) below. This shows that the NumP *dwa jabłka* 'two apples' receives its case from *po*, as otherwise it would bear the genitive of negation, as in (11b).

(11) a. Nie dałem im po dwa/*dwóch jabłka/*jabłek.
NEG gave-I them.DAT DISTR two.ACC/*GEN apples.ACC/*GEN
'I didn't give them two apples (each).'

b. Nie dałem im dwóch/*dwa jabłek/*jabłka.
NEG gave-I them.DAT two.GEN/*ACC apples.GEN/*ACC
'I didn't give them two apples.'

Finally, (10) illustrates that *po* sometimes does not assign case and may be transparent to case assignment; the dative on *trzem osobom* 'three people' is assigned by the verb. While similar examples may also be found for other morphological cases, including instrumental, genitive and locative, they are often judged marginal or downright unacceptable, which shows that the availability of this third lexeme *po* is restricted.

Despite such morphosyntactic idiosyncrasies, Przepiórkowski 2013 in the HPSG settings and Przepiórkowski and Patejuk 2013 within LFG, provide a unified analysis of the three lexemes *po* which treats all of them as heads. Hence, in the remainder of this paper we will not distinguish them and we will assume that the phrase *po* combines with is its object.

[4] The first two examples, (8)–(9), are constructed but uncontroversial. As mentioned below, the acceptability status of examples such as (10) is disputed, so this example is attested; NKJP stands for *Narodowy Korpus Języka Polskiego* 'National Corpus of Polish' (http://nkjp.pl/; Przepiórkowski et al. 2012). Henceforth, Polish examples will not be explicitly marked as such.

Polish patterns with German rather than English in allowing the distributive share in the subject position. In a classic paper on *po*, Łojasiewicz (1979:154) cites the following examples with (post-verbal) subjects:[5]

(12) Z drzew spadło po jabłku.
from trees fell DISTR apple.LOC
'An apple fell from each tree.'

(13) W pokojach będą po dwa fotele.
in rooms be.FUT DISTR two armchairs
'There will be two armchairs in each room.'

Such cases pose no problem for the analysis proposed below.

One aspect of distance distributivity in Polish that is not considered here is the possibility of distribution over events. The argument that distributive elements like the German *jeweils* may quantify over events comes from examples such as (14) adduced by Moltmann (1997) and cited in Zimmermann 2002:28:

(14) Peter hat Maria aus jeweils zwei Gründen kritisiert und gelobt. (German)
Peter has Maria for DISTR two reasons criticised and praised
'Peter has criticised and praised Maria for two reasons respectively.'

This sentence means that for each of the two events involving Peter as an agent and Maria as a patient, namely, that of criticising and that of praising, Peter had two reasons to be so involved in them. Similarly, the only way to interpret (15), also from Zimmermann 2002:36, is to assume a contextually given set of events of the Pope's travels that *jeweils* quantifies over.

(15) Der Papst ist in jeweils drei Länder gefahren. (German)
the Pope has to DISTR three countries travelled
'The Pope has travelled to three countries each.'

Similar examples can be constructed in Polish:

(16) Piotr miał po dwa powody by chwalić i krytykować Marię.
Piotr had DISTR two reasons to praise and criticise Maria.
'Peter had two reasons each to criticise and to praise Maria.'

(17) Papież zwiedzał po trzy kraje.
Pope visited DISTR three countries
'The Pope visited three countries each time.'

Nevertheless, we assume simplistic eventless representations here and do not treat such cases of distributivity over events.[6]

[5] The case of *dwa fotele* 'two armchairs' is not given in (13), as it is not clear whether this phrase occurs in the nominative or in the accusative here; Przepiórkowski 2013 and Przepiórkowski and Patejuk 2013 argue for the accusative, despite appearances to the contrary.

[6] In Przepiórkowski 2014a, we show that the extension of the current analysis to distribution over events is immediate.

3 Glue Semantics

In traditional approaches to compositionality (e.g. Heim and Kratzer 1998), meanings combine when they are expressed by siblings in a constituency tree. By contrast, in Glue Semantics (Dalrymple 1999, 2001) coupled with Lexical-Functional Grammar (Bresnan 2001, Dalrymple 2001), meanings combine based on f(unctional)-structures, rather than on c(onstituent)-structures, and meaning representations are paired with glue formulae specifying how these meanings combine with which other meanings. Any pair consisting of a meaning representation and a glue formula is called a *meaning constructor*.

For example, the glue part of the meaning constructor for various forms of *yawn* is:

(18) $e((\uparrow \textsc{subj})) \multimap t(\uparrow)$

We follow here the First Order approach to Glue Semantics (Kokkonidis 2008), where glue formulae contain parameterised types, and assume two basic type constructors: *e* (for *entity*) and *t* (for *truth*). The parameters of such basic type constructors are f-structures. As usual in LFG, the up arrow ↑ in a lexical entry denotes the f-structure of the word, so (↑ SUBJ) – with obligatory parentheses, hence the double parentheses in the antecedent of (18) – denotes the f-structure of the subject of this word. In effect, (18) says that by consuming the *e* type corresponding to the subject of a form of *yawn* such as *yawned*, we may produce the *t* type corresponding to *yawned* and, hence, to the whole clause headed by *yawned* (in LFG heads normally share their f-structure with their projections).

This mode of composition remains true regardless of specific tree configurations. For example, when *yawn* is a complement of a control verb, its covert subject is never realised in the c(onstituent)-structure, according to standard LFG analyses, but it is still present in its f-structure, as the value of the SUBJ attribute, so (18) is still relevant.

Glue Semantics is resource-sensitive: once a semantic resource – i.e., a glue formula – is consumed, it cannot be reused. Dually, all semantic resources introduced by lexical items (or otherwise; semantic resources may be introduced constructionally) must be consumed in a derivation of the semantic resource of the whole sentence. For example, assuming that *David* introduces a glue formula matching the antecedent of ⊸ in (18), a proof rule analogous to *modus ponens* (and introduced more formally below) consumes both formulae and produces the formula $t(\uparrow)$ for the sentence *David yawned.* As this is the only resource left, the proof succeeds.

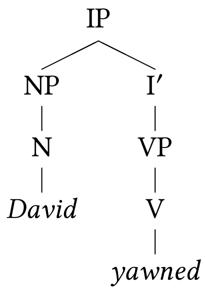
The other part of the meaning constructor is a formula in any language that allows application and abstraction such as the language of the first-order predicate logic with lambda calculus. For example, the meaning of *David* can be defined as a logical constant, *David*, and the meaning of *yawned* can be defined as usual, as $\lambda X.yawn(X)$ (ignoring event variables, semantic roles, tense and aspect, etc.). In complete meaning constructors, the meaning part is separated from the glue part by the uninterpreted colon character (:), so the complete meaning constructors for *David* and *yawned* are as in the second lines of the following lexical entries:

(19) *David* N (↑ PRED) = 'DAVID'
$David : e(\uparrow)$

(20) *yawned* V (↑ PRED) = 'YAWN<SUBJ>'
$\lambda X.yawn(X) : e((\uparrow \textsc{subj})) \multimap t(\uparrow)$

According to these lexical entries and standard LFG constituency rules, *David yawned* receives the c-structure displayed in (21) and the f-structure in (22); moreover, given this f-structure, meaning constructors are instantiated as in (23):[7]

(21)



(22) $\boxed{0}\begin{bmatrix} \text{PRED} & \text{‘YAWN}\langle\boxed{1}\rangle\text{’} \\ \text{SUBJ} & \boxed{1}\begin{bmatrix}\text{PRED} & \text{‘DAVID’}\end{bmatrix} \end{bmatrix}$

(23) **[David]** $David : e(\boxed{1})$
[yawned] $\lambda X.yawn(X) : e(\boxed{1}) \multimap t(\boxed{0})$

Now, using one of the proof rules of Glue Semantics, namely, the Implication Elimination rule in (24), and performing the usual β-reduction, the meaning of *David yawned* may be derived from the meaning constructors in (23) as shown in (25):

(24) $$\frac{a : A \qquad f : A \multimap B}{f(a) : B} \multimap_{\mathcal{E}}$$

(25) $$\frac{David : e(\boxed{1}) \qquad \lambda X.yawn(X) : e(\boxed{1}) \multimap t(\boxed{0})}{yawn(David) : t(\boxed{0})} \multimap_{\mathcal{E}}$$

Since both meaning resources introduced by lexical items, $e(\boxed{1})$ and $e(\boxed{1}) \multimap t(\boxed{0})$, are consumed in this proof, and the only meaning resource produced, $t(\boxed{0})$, corresponds to the f-structure of the whole sentence, this is a valid proof that the meaning side of the whole sentence is $yawn(David)$.

Obviously, we cannot do justice to Glue Semantics within the confines of this paper; the above is only meant to make the analysis below more accessible to motivated readers not familiar with this approach. The best introduction to Glue Semantics may still be found in the classical LFG textbook of Dalrymple 2001, on which the above exposition is based. Early influential papers are gathered in Dalrymple 1999, but they may be a little hard for an uninitiated reader, as they use a different – perhaps less transparent – notation; the exception is Dalrymple et al. 1999a, which introduces the notation adopted in subsequent work on Glue Semantics. As mentioned above, in this paper we assume the First Order approach to Glue advocated in Kokkonidis 2008, which allows quantification over e types, not just over t types, as in previous versions of Glue Semantics – the analysis proposed below crucially relies on this type of quantification.

The glue side of meaning constructors is a fragment of linear logic (Girard 1987). Resources are understood here as types parameterised with functional structures, but that does not mean that Glue Semantics is necessarily tightly coupled with LFG; versions of this approach have been proposed for other grammatical formalisms, including Head-driven Phrase Structure Grammar (Asudeh and Crouch 2002) and Lexicalized Tree-Adjoining Grammar (Frank and van Genabith 2001). Also, while the meaning side adopted here is a version of the language of predicate logic with lambdas, this is not a necessity. Instead, Intensional Logic is employed in Dalrymple et al.

[7]We adopt here the HPSG convention of naming feature structures with boxed numbers and of signalling structure-sharing by the repeated occurrence of a boxed number (cf. $\boxed{1}$ in (22)). Labels of meaning constructors are written in **[bold-in-square-brackets]**.

1999c and various derivatives of Discourse Representation Theory are used in Dalrymple et al. 1999b, Crouch and van Genabith 1999, and more recently in Haug 2013.

4 Analysis

4.1 Preliminaries

Let us first consider the two run-of-the-mill examples below:

(26) Chłopcy mają po dwa tatuaże.
boys.NOM have.PL DISTR two.ACC tattoos.ACC
'(The/Some) boys have two tattoos each.'

(27) Piotr kupił dziewczynom po róży.
Piotr.NOM bought.SG girls.DAT DISTR rose.LOC
'Peter bought (the/some) girls a rose each.'

In both examples the *po*-phrase (the distributive share) occupies the position of the direct object of the verb; the purely morphosyntactic difference between the accusative case of *dwa tatuaże* 'two tattoos' in (26) and the locative case of *róży* 'rose' in (27) was explained in section 2. The sorting key is expressed by the subject *Chłopcy* 'boys' in (26) and by the indirect object *dziewczynom* 'girls' in (27).

The intended meaning representations of these two examples are given below:

(28) Intended meaning representation of (26):
$exists(Z, boy^s(Z) \wedge |Z| > 1,$
$all(X, |X| = 1 \wedge X \subset Z,$
$exists(V, |V| = 2 \wedge tattoo^s(V), have(X, V))))$

(29) Intended meaning representation of (27):
$exists(Z, girl^s(Z) \wedge |Z| > 1,$
$all(X, |X| = 1 \wedge X \subset Z,$
$exists(V, |V| = 1 \wedge rose^s(V), bought(p, V, X))))$

In fact, both examples taken out of context are similarly ambiguous: the plural bare NPs (*Chłopcy* 'boys' and *dziewczynom* 'girls') may be interpreted either as indefinites or as definites. For reasons of simplicity, both indefinites and definites are represented as generalised quantifiers in the current paper; the former are approximated by the existential quantifier *exists*, as in the representations above, and the latter will be represented below via the *iota* relation.

As common in LFG and Glue Semantics, generalised quantifiers are represented here as relations between an individual and two propositions involving that individual, so that *Everyone yawned* has the representation $all(X, person(X), yawn(X))$ (Dalrymple 2001:227). Moreover, we follow Dotlačil 2012 and earlier work on treating entities as sets,[8] and properties – as sets of such sets. For example, boy^s is the property of being a non-empty set of boys – either a singleton or a set of higher cardinality (the superscript s indicates the possible plural) – and $\lambda Z. |Z| > 1 \wedge boy^s(Z)$ is the property of being a set of at least two boys. On this view, the standard inclusion relation $\subseteq$ is defined on entities.

[8] In particular, we do not distinguish between singleton sets and their elements.

How do these meaning representations differ from meanings of corresponding examples without the distributive element? The relevant examples and their intended collective meanings (assuming the existential closure of all bare NPs) are given below.[9]

(30) a. Chłopcy mają dwa tatuaże. (Cf. (26))
boys.NOM have.PL two.ACC tattoos.ACC
'(The/Some) boys have two tattoos.'

b. $exists(Z, boy^s(Z) \wedge |Z| > 1,$ (Cf. (28))
$exists(V, |V| = 2 \wedge tattoo^s(V), have(Z, V))))$

(31) a. Piotr kupił dziewczynom różę. (Cf. (27))
Piotr.NOM bought.SG girls.DAT rose.ACC
'Peter bought a rose for (the/some) girls.'

b. $exists(Z, girl^s(Z) \wedge |Z| > 1,$ (Cf. (29))
$exists(V, |V| = 1 \wedge rose^s(V), bought(p, V, Z))))$

The difference between the meaning representations in (30b) and (31b) above and the earlier representations in (28) and (29) should make the impact of the distributive *po* clear: it takes a property holding of some set and transforms it into an analogous property holding of each singleton subset of the set. We formalise this observation in the following subsection.

4.2 *Semantics of* po *and worked-out example*

The first version of the meaning constructor for *po*, labelled as **[distr]**, is given below:[10]

(32) **[distr]** $\lambda S.\lambda Z.all(X, |X| = 1 \wedge X \subset Z, S(X)) : \forall G, H.\ [e(G) \multimap t(H)] \multimap [e(G) \multimap t(H)]$

The meaning part (on the left of the colon) directly reflects the considerations of the previous subsection: *po* takes a property S and returns a property that holds of Z if and only if S holds of all singleton (proper) subsets of Z. The glue part (on the right of the colon) says that *po* is an identity function on semantic resources corresponding to properties: it consumes a resource $[e(G) \multimap t(H)]$ (for any G and H) in order to produce the same resource. Hence, *po* as construed above may combine with just any $\langle e, t \rangle$ property in the sentence; as we will see below, this analysis is too permissive and will be further constrained in section 4.4.

We will illustrate the analysis in detail on the basis of example (26), repeated below (with the additional assumption that the subject is to be understood existentially):

(26′) Chłopcy mają po dwa tatuaże.
boys.NOM have.PL DISTR two.ACC tattoos.ACC
'Some boys have two tattoos each.'

As usual in LFG and Glue Semantics, the two common nouns occurring in this sentence have the following lexical entries (ignoring morphosyntactic features such as case or gender):

[9] In case of (30), the collective meaning may be difficult to get, unless one understands tattoos as temporary sticker tattoos (before they are applied).

[10] The meaning side is essentially the semantic representation of the abstract DIST(ributivity) operator proposed by Link 1991. The arguments given by Zimmermann 2002:68–69 that the German *jeweils* is not an overt realisation of DIST do not bear on the choice of this meaning representation here.

(33) *chłopcy* N $(\uparrow \textsc{pred}) = \text{‘}\textsc{boys}\text{’}$
$\lambda X.boy^s(X) \wedge |X| > 1 : e(\uparrow) \multimap t(\uparrow)$

(34) *tatuaże* N $(\uparrow \textsc{pred}) = \text{‘}\textsc{tattoos}\text{’}$
$\lambda X.tattoo^s(X) \wedge |X| > 1 : e(\uparrow) \multimap t(\uparrow)$

Simplifying somewhat, we treat cardinals as existential quantifiers:

(35) *dwa* Num $(\uparrow \textsc{spec}) = 2$
$\lambda R.\lambda S.exists(Y, |Y| = 2 \wedge R(Y), S(Y)) :$
$\forall H.\ [e(\uparrow) \multimap t(\uparrow)] \multimap [[e(\uparrow) \multimap t(H)] \multimap t(H)]$

While there are syntactic arguments that numerals take the following NPs as complements, that is, that phrases of the form Num+NP are really headed by the numeral, we simplify here by treating the numeral and the following noun as co-heads. Given the c-structure rule in (36), we get the f-structure for *dwa tatuaże* ‘two tattoos’ shown in (37):

(36) NumP $\rightarrow$ Num N
$\uparrow{=}\downarrow$ $\uparrow{=}\downarrow$

(37) $\boxed{3}\begin{bmatrix} \textsc{spec} & \text{‘2’} \\ \textsc{pred} & \text{‘}\textsc{tattoos}\text{’} \end{bmatrix}$

Given this f-structure, all occurrences of $\uparrow$ in (34) and in (35) instantiate to $\boxed{3}$, so we can construct the following proof for the meaning of *dwa tatuaże* ‘two tattoos’:[11]

(38)
$$\frac{\begin{array}{l}\lambda R.\lambda S.exists(Y, |Y| = 2 \wedge R(Y), S(Y)) : \\ \forall H.\ [e(\boxed{3}) \multimap t(\boxed{3})] \multimap [[e(\boxed{3}) \multimap t(H)] \multimap t(H)]\end{array} \qquad \begin{array}{l}\lambda X.tattoo^s(X) \wedge |X| > 1 : \\ e(\boxed{3}) \multimap t(\boxed{3})\end{array}}{\begin{array}{l}\lambda S.exists(Y, |Y| = 2 \wedge tattoo^s(Y), S(Y)) : \\ \forall H.[e(\boxed{3}) \multimap t(H)] \multimap t(H)\end{array}} \multimap_{\mathcal{E}}$$

The only missing lexical entries needed to analyse (26) are that of the main verb, *mają* ‘have’, as in (39), and that of *po*, as in (40):

(39) *mają* V $(\uparrow \textsc{pred}) = \text{‘}\textsc{have}\langle\textsc{subj,obj}\rangle\text{’}$
$\lambda X.\lambda Y.have(X, Y) : e((\uparrow \textsc{subj})) \multimap [e((\uparrow \textsc{obj})) \multimap t(\uparrow)]$

(40) *po* P $(\uparrow \textsc{pred}) = \text{‘}\textsc{po}\langle\textsc{obj}\rangle\text{’}$
$\lambda P.P : \forall F.\ [e(\uparrow) \multimap t(F)] \multimap [e((\uparrow \textsc{obj})) \multimap t(F)]$
$\lambda S.\lambda Z.all(X, |X| = 1 \wedge X \subset Z, S(X)) : \forall G, H.\ [e(G) \multimap t(H)] \multimap [e(G) \multimap t(H)]$

The lexical entry of the verb should be self-explanatory at this stage: the semantic resources of the subject and the object must be consumed to produce a semantic resource corresponding to the verb (and, hence, to the whole sentence headed by this verb). On the other hand, the preposition *po*[12] introduces two meaning constructors. The effect of the first one is that whatever property P is specified elsewhere to hold of the meaning of the *po*-phrase, it must hold of the meaning of the object of *po* instead. The other one is **[distr]** discussed above. These lexical entries, together with standard c-structure rules, produce the following f-structure for the complete sentence in (26):

[11] Each meaning constructor is broken into two lines for typographical reasons. We also drop the conjunct $|Y| > 1$ in the conclusion, as it follows from $|Y| = 2$.

[12] As discussed in section 2, there are three different lexemes *po* in Polish, but they are all analysed as heads, so the lexical entry in (40) is a sufficiently good approximation of all of them.

(41)
$$\boxed{0}\left[\begin{array}{ll} \text{PRED} & \text{`HAVE}\langle\boxed{1},\boxed{2}\rangle\text{'} \\ \text{SUBJ} & \boxed{1}\left[\begin{array}{ll}\text{PRED} & \text{`BOYS'}\end{array}\right] \\ \text{OBJ} & \boxed{2}\left[\begin{array}{ll}\text{PRED} & \text{`PO}\langle\boxed{3}\rangle\text{'}\\ \text{OBJ} & \boxed{3}\left[\begin{array}{ll}\text{SPEC} & \text{`2'}\\ \text{PRED} & \text{`TATTOOS'}\end{array}\right]\end{array}\right]\end{array}\right]$$

Given this f-structure, the meaning of *mają* ‘have’ instantiates to (42) and the first meaning constructor of *po* instantiates to (43):

(42) **[have]** $\lambda X.\lambda Y.have(X,Y) : e(\boxed{1}) \multimap [e(\boxed{2}) \multimap t(\boxed{0})]$

(43) **[po]** $\lambda P.P : \forall F.[e(\boxed{2}) \multimap t(F)] \multimap [e(\boxed{3}) \multimap t(F)]$

At this point another Glue Semantics proof rule is needed, Implication Introduction, which says that if the introduction of an assumption $[x : A]$ leads to a proof of $f : B$ then $\lambda x.f : A \multimap B$ is proved:

(44)
$$\dfrac{\begin{array}{c}[x:A]^1\\ \vdots \\ f:B\end{array}}{\lambda x.f : A \multimap B}\multimap_{\mathcal{I},1}$$

Using this rule, (45) may be proved from (42) and (43) as shown in (46):

(45) **[have-po]** $\lambda X.\lambda Y.have(X,Y) : e(\boxed{1}) \multimap [e(\boxed{3}) \multimap t(\boxed{0})]$

(46)
$$\dfrac{\dfrac{\dfrac{[X:e(\boxed{1})]^1 \quad \begin{array}{l}\lambda X.\lambda Y.have(X,Y):\\ e(\boxed{1})\multimap[e(\boxed{2})\multimap t(\boxed{0})]\end{array}}{\lambda Y.have(X,Y):e(\boxed{2})\multimap t(\boxed{0})}\multimap_{\mathcal{E}} \quad \lambda P.P:\forall F.[e(\boxed{2})\multimap t(F)]\multimap[e(\boxed{3})\multimap t(F)]}{\lambda Y.have(X,Y):e(\boxed{3})\multimap t(\boxed{0})}\multimap_{\mathcal{E}}}{\lambda X.\lambda Y.have(X,Y):e(\boxed{1})\multimap[e(\boxed{3})\multimap t(\boxed{0})]}\multimap_{\mathcal{I},1}$$

The conclusion may be combined with the conclusion of proof (38), repeated in (47), to render the meaning of *mają dwa tatuaże* ‘have two tattoos’ in (48); the proof is shown in (49):

(47) **[two-tattoos]** $\lambda S.exists(Y, |Y| = 2 \wedge tattoo^s(Y), S(Y)) : \forall H.\ [e(\boxed{3}) \multimap t(H)] \multimap t(H)$

(48) **[have-po-two-tattoos]** $\lambda X.exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X,Y)) :\ e(\boxed{1}) \multimap t(\boxed{0})$

(49)
$$\dfrac{\dfrac{\dfrac{[X:e(\boxed{1})]^1 \quad \begin{array}{l}\lambda X.\lambda Y.have(X,Y):\\ e(\boxed{1})\multimap[e(\boxed{3})\multimap t(\boxed{0})]\end{array}}{\lambda Y.have(X,Y):e(\boxed{3})\multimap t(\boxed{0})}\multimap_{\mathcal{E}} \quad \begin{array}{l}\lambda S.exists(Y,|Y|=2\wedge tattoo^s(Y),S(Y)):\\ \forall H.\ [e(\boxed{3})\multimap H]\multimap t(H)\end{array}}{exists(Y,|Y|=2\wedge tattoo^s(Y),have(X,Y)):t(\boxed{0})}\multimap_{\mathcal{E}}}{\lambda X.exists(Y,|Y|=2\wedge tattoo^s(Y),have(X,Y)):\ e(\boxed{1})\multimap t(\boxed{0})}\multimap_{\mathcal{I},1}$$

The conclusion of proof (49) is of the form that may be combined with the second meaning constructor for *po* given in (40):

(50)
$$\dfrac{\begin{array}{l}\lambda X.exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X,Y)) : \\ e(\boxed{1}) \multimap t(\boxed{0})\end{array} \quad \begin{array}{l}\lambda S.\lambda Z.all(X, |X| = 1 \wedge X \subset Z, S(X)) : \\ \forall G, H.\ [e(G) \multimap t(H)] \multimap [e(G) \multimap t(H)]\end{array}}{\begin{array}{l}\lambda Z.all(X, |X| = 1 \wedge X \subset Z, exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X,Y))) : \\ e(\boxed{1}) \multimap t(\boxed{0})\end{array}} \multimap_{\mathcal{E}}$$

Now we face an apparent problem, as – apart from the resource in the conclusion of proof (50) – the only other resource left is that of *chłopcy* 'boys', introduced in (33) and instantiated here to (51), and these two resources are incompatible (cannot be combined).

(51) **[boys]** $\lambda X.boy^s(X) \wedge |X| > 1 : e(\boxed{1}) \multimap t(\boxed{1})$

However, as noted above, such bare NPs are understood as either indefinites or as definites, so the grammar must provide appropriate meaning constructors completing the lexical meanings of bare NPs. As it is not the aim of this paper to investigate the representation of (in)definites, we approximate them via generalised quantifiers (even though it is well known that they have different scopal properties than usual quantifiers). In the case at hand, the meaning constructor that is needed is (compare this to the meaning of *dwa* 'two' in (35)):

(52) **[existential]** $\lambda R.\lambda S.exists(Z, R(Z), S(Z))$:
$\forall H.\ [e(\boxed{1}) \multimap t(\boxed{1})] \multimap [[e(\boxed{1}) \multimap t(H)] \multimap t(H)]$

Once this constructor is available, the existential meaning of *chłopcy* 'boys' may be derived using the Implication Elimination proof rule:

(53)
$$\dfrac{\begin{array}{l}\lambda X.boy^s(X) \wedge |X| > 1 : \\ e(\boxed{1}) \multimap t(\boxed{1})\end{array} \quad \begin{array}{l}\lambda R.\lambda S.exists(Z, R(Z), S(Z)) : \\ \forall H.\ [e(\boxed{1}) \multimap t(\boxed{1})] \multimap [[e(\boxed{1}) \multimap t(H)] \multimap t(H)]\end{array}}{\begin{array}{l}\lambda S.exists(Z, boy^s(Z) \wedge |Z| > 1, S(Z)) : \\ \forall H.[e(\boxed{1}) \multimap t(H)] \multimap t(H)\end{array}} \multimap_{\mathcal{E}}$$

Applying the same proof rule to the conclusions of (50) and (53), we obtain the same (up to variable names) meaning side as the intended meaning representation of (26), given in (28):

(54)
$$\dfrac{\begin{array}{l}\lambda Z.all(X, |X| = 1 \wedge X \subset Z, \\ exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X,Y))) : \\ e(\boxed{1}) \multimap t(\boxed{0})\end{array} \quad \begin{array}{l}\lambda S.exists(Z, boy^s(Z) \wedge |Z| > 1, S(Z)) : \\ \forall H.[e(\boxed{1}) \multimap t(H)] \multimap t(H)\end{array}}{\begin{array}{l}exists(Z, boy^s(Z) \wedge |Z| > 1, \\ \quad all(X, |X| = 1 \wedge X \subset Z, \\ \quad\quad exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X,Y)))) : t(\boxed{0})\end{array}} \multimap_{\mathcal{E}}$$

The schematic structure of the whole proof is given below, with references to subproofs:

(55)
$$\dfrac{\dfrac{\dfrac{\dfrac{\textbf{[have] [po]}}{\textbf{[have-po]}}(46) \quad \dfrac{\textbf{[two] [tattoos]}}{\textbf{[two-tattoos]}}(38)}{\textbf{[have-po-two-tattoos]}}(49) \quad \textbf{[distr]}}{\textbf{[distr-have-po-two-tattoos]}}(50) \quad \dfrac{\textbf{[boys] [existential]}}{\textbf{[boys-existential]}}(53)}{\textbf{[boys-existential-distr-have-po-two-tattoos]}}(54)$$

Note that all resources introduced by lexical items have been consumed in the process and that the only resource left is $t(\boxed{0})$, which corresponds to the complete sentence; hence, this is a linguistically valid proof (Asudeh 2012:chap. 5).

An analogous proof could be constructed for the definite reading of *chłopcy* 'boys', using the following meaning constructor instead of **[existential]** of (52):

(56) **[definite]** $\lambda R.\lambda S.iota(Z, R(Z), S(Z))$:
$\forall H.\, [e(\boxed{1}) \multimap t(\boxed{1})] \multimap [[e(\boxed{1}) \multimap t(H)] \multimap t(H)]$

Such meaning constructors must be optionally available for any common noun. If the noun contributes to the restriction of a lexical quantifier, as in case of *tatuaże* 'tattoos' restricting the quantifier *dwa* 'two', optional meaning constructors of this kind cannot be used – the lexical quantifier consumes the resources necessary to activate such constructors. On the other hand, when there is no appropriate lexical quantifier, either the existential closure or the definiteness meaning constructor may activate and combine with the bare noun.[13]

4.3 Sorting key within distributive share

Let us now turn to (5), repeated below as (5′), where the sorting key, *25 krajów* '25 countries', is syntactically embedded within the phrase expressing the distributive share, *po 3 przedstawicieli 25 krajów* '3 representatives of (each of) 25 countries'; the schematic constituent structure is repeated as (6′).

(5′) Przybyło po 3 przedstawicieli 25 krajów.
arrive.PAST DISTR 3 representatives 25.GEN countries.GEN
'3 representatives arrived from each of 25 countries.'

(6′) Przybyło [po [3 [przedstawicieli [25 krajów]]]].

Lexical entries for *3* and *25* parallel that for *dwa* 'two' given in (35):

(57) *3* Num (↑ SPEC) = 3
$\lambda R.\lambda S.exists(Y, |Y| = 3 \wedge R(Y), S(Y))$:
$\forall H.\, [e(\uparrow) \multimap t(\uparrow)] \multimap [[e(\uparrow) \multimap t(H)] \multimap t(H)]$

(58) *25* Num (↑ SPEC) = 25
$\lambda R.\lambda S.exists(Y, |Y| = 25 \wedge R(Y), S(Y))$:
$\forall H.\, [e(\uparrow) \multimap t(\uparrow)] \multimap [[e(\uparrow) \multimap t(H)] \multimap t(H)]$

Similarly, the lexical entry for *krajów* 'countries' is analogous to those for *chłopcy* 'boys' and *tatuaże* 'tattoos' in (33) and (34), and the entry for *przybyło* 'arrived' is simpler than that for *mają* 'have' in (39), as it only takes one argument:

(59) *krajów* N (↑ PRED) = 'COUNTRIES'
$\lambda X.country^{s}(X) \wedge |X| > 1 : e(\uparrow) \multimap t(\uparrow)$

(60) *przybyło* V (↑ PRED) = 'ARRIVE<SUBJ>'
$\lambda X.arrive(X) : e((\uparrow \text{SUBJ})) \multimap t(\uparrow)$

[13]We assume that such optional meaning constructors are introduced in lexical entries of common nouns, as part of a common noun template, so as to avoid missing generalisations (Asudeh et al. 2013); another option would be to add them to appropriate c-structure rules.

What is new in this example is a relational noun, *przedstawicieli* ‘representatives’:[14]

(61) *przedstawicieli* N $(\uparrow \textsc{pred}) = \text{‘REPRESENTATIVES<OBJ>’}$
$\lambda Y.\lambda X.representative^{s}(X,Y) \wedge |X| > 1 :$
$e((\uparrow \textsc{obj})) \multimap [e(\uparrow) \multimap t(\uparrow)]$

The meaning constructor of (61) differs from that of (59) and other non-relational nouns in the additional requirement of the argument of the noun.

With these lexical entries, as well as the lexical entry for *po* given in (40) above, the f-structure of (5) is as shown in (62).

(62)
$$\boxed{0}\left[\begin{array}{ll} \textsc{pred} & \text{‘ARRIVED}\langle\boxed{1}\rangle\text{’} \\ \textsc{subj} & \boxed{1}\left[\begin{array}{ll} \textsc{pred} & \text{‘PO}\langle\boxed{2}\rangle\text{’} \\ \textsc{obj} & \boxed{2}\left[\begin{array}{ll} \textsc{spec} & \text{‘3’} \\ \textsc{pred} & \text{‘REPRESENTATIVE}\langle\boxed{3}\rangle\text{’} \\ \textsc{obj} & \boxed{3}\left[\begin{array}{ll} \textsc{spec} & \text{‘25’} \\ \textsc{pred} & \text{‘COUNTRY’} \end{array}\right] \end{array}\right] \end{array}\right] \end{array}\right]$$

The intended meaning of (5), given in (63), may be attained via the proof schematically shown in (64), where the particular meaning constructors, as instantiated for (62), are given in (65)–(76).[15]

(63) $exists(Z, |Z| = 25 \wedge country^{s}(Z),$
$all(X, |X| = 1 \wedge X \subset Z,$
$exists(V, |V| = 3 \wedge representative^{s}(V,X), arrived(V)))) : t(\boxed{0})$

(64)
$$\dfrac{\dfrac{\dfrac{\dfrac{\textbf{[arrived] [po]}}{\textbf{[arrived-po]}}\multimap_{\mathcal{E}} \quad \dfrac{\textbf{[3] [representatives]}}{\textbf{[3-representatives]}}\multimap_{\mathcal{EEI}}}{\textbf{[arrived-po-3-representatives]}}\multimap_{\mathcal{EEI}} \quad \textbf{[distr]}}{\textbf{[distr-arrived-po-3-representatives]}}\multimap_{\mathcal{E}} \quad \dfrac{\textbf{[25] [countries]}}{\textbf{[25-countries]}}\multimap_{\mathcal{E}}}{\textbf{[25-countries-distr-arrived-po-3-representatives]}}\multimap_{\mathcal{E}}$$

(65) **[25]**
$\lambda R.\lambda S.exists(X, |X| = 25 \wedge R(X), S(X)) : \forall H.\ [e(\boxed{3}) \multimap t(\boxed{3})] \multimap [[e(\boxed{3}) \multimap t(H)] \multimap t(H)]$

(66) **[countries]**
$\lambda X.country^{s}(X) \wedge |X| > 1 : e(\boxed{3}) \multimap t(\boxed{3})$

(67) **[25-countries]**
$\lambda S.exists(X, |X| = 25 \wedge country^{s}(X), S(X)) : \forall H.[e(\boxed{3}) \multimap t(H)] \multimap t(H)$

(68) **[3]**
$\lambda R.\lambda S.exists(X, |X| = 3 \wedge R(X), S(X)) : [\forall H.\ [e(\boxed{2}) \multimap t(\boxed{2})] \multimap [[e(\boxed{2}) \multimap t(H)] \multimap t(H)]$

[14]We remain agnostic as to whether OBJ, assumed in (61), is really the right grammatical function for the complement of *przedstawicieli* ‘representatives’. Dalrymple et al. 1999c:57 and Dalrymple 2001:249 analyse arguments of English nouns *rumor* and *relative*, introduced by the prepositional markers *about* and *of*, as values of $\textsc{obl}_{\textsc{about}}$ and $\textsc{obl}_{\textsc{of}}$, respectively.

[15]The parts of the proof marked with $\multimap_{\mathcal{EEI}}$ consist of three steps analogous to subproofs given in (46) and in (49). Again, we omit $|X| > 1$ once it follows from particular cardinalities contributed by the numerals.

(69) **[representatives]**
$\lambda Y.\lambda X representative^s(X,Y) \wedge |X| > 1 : e(\boxed{3}) \multimap [e(\boxed{2}) \multimap t(\boxed{2})]$

(70) **[3-representatives]**
$\lambda Y.\lambda S.exists(X, |X| = 3 \wedge representative^s(X,Y), S(X)) : \forall H.\ e(\boxed{3}) \multimap [[e(\boxed{2}) \multimap t(H)] \multimap t(H)]$

(71) **[po]**
$\lambda P.P : \forall F.\ [e(\boxed{1}) \multimap t(F)] \multimap [e(\boxed{2}) \multimap t(F)]$

(72) **[arrived]**
$\lambda X.arrived(X) : e(\boxed{1}) \multimap t(\boxed{0})$

(73) **[arrived-po]**
$\lambda X.arrived(X) : e(\boxed{2}) \multimap t(\boxed{0})$

(74) **[arrived-po-3-representatives]**
$\lambda Y.exists(X, |X| = 3 \wedge representative^s(X,Y), arrived(X)) : e(\boxed{3}) \multimap t(\boxed{0})$

(75) **[distr-arrived-po-3-representatives]** (see (32) for **[distr]**)
$\lambda Z.all(X, |X| = 1 \wedge X \subset Z,$
$\quad exists(V, |V| = 3 \wedge representative^s(V,X), arrived(V))) : e(\boxed{3}) \multimap t(\boxed{0})$

(76) **[25-countries-distr-arrived-po-3-representatives]**
$exists(Z, |Z| = 25 \wedge country^s(Z),$
$\quad all(X, |X| = 1 \wedge X \subset Z,$
$\quad\quad exists(V, |V| = 3 \wedge representative^s(V,X), arrived(V)))) : t(\boxed{0})$

This proof shows that the analysis proposed in the previous subsection provides a correct meaning representation for troublesome cases when the sorting key is embedded within the phrase expressing the distributive share.

4.4 Constraining analysis

Unfortunately, as it stands, the analysis heavily overgenerates. For example, apart from (64), there are other proofs for the same sentence, leading to nonsensical or wrong meaning representations. The problem is that the meaning of *po*, as given in (32) and (40), may combine with any (appropriately typed) property available in the derivation, e.g., with **[countries]** in (66), with **[arrived]** in (72) or with the property derived from **[representatives]** in (69) by introducing the assumption $Y : e(\boxed{3})$ and using the Implication Elimination rule (24).

We will illustrate this problem with a simpler example, by showing that the sentence *Chłopcy mają po dwa tatuaże* '(Some/The) boys have two tattoos each', given as (26) in section 4.2, has another proof, leading to the incorrect meaning in (77), paraphrased as "for each of some two tattoos, there are some boys that have it."

(77) $exists(Y, |Y| = 2 \wedge tattoo^s(Y),$
$\quad all(X, |X| = 1 \wedge X \subset Y,$
$\quad\quad exists(Z, boy^s(Z) \wedge |Z| > 1, have(Z,X))))$

The proof is analogous to (55), and it is given in (78) below, with references to subproofs:

(78)
$$\dfrac{\dfrac{\dfrac{\textbf{[have] [po]}}{\textbf{[have-po]}}\,(46) \quad \dfrac{\textbf{[boys] [existential]}}{\textbf{[boys-existential]}}\,(53)}{\textbf{[boys-existential-have-po]}}\,(79) \quad \textbf{[distr]}}{\textbf{[distr-boys-existential-have-po]}}\,(80) \quad \dfrac{\textbf{[two] [tattoos]}}{\textbf{[two-tattoos]}}\,(38)$$
$$\overline{\textbf{[two-tattoos-distr-boys-existential-have-po]}}\,(81)$$

(79)
$$\dfrac{\dfrac{\dfrac{\dfrac{[Y : e(\boxed{3})]^2 \quad \dfrac{[X : e(\boxed{1})]^1 \quad \begin{array}{c}\lambda X.\lambda Y.\mathit{have}(X,Y) : \\ e(\boxed{1}) \multimap [e(\boxed{3}) \multimap t(\boxed{0})]\end{array}}{\lambda Y.\mathit{have}(X,Y) : e(\boxed{3}) \multimap t(\boxed{0})}\multimap_{\mathcal{E}}}{\mathit{have}(X,Y) : t(\boxed{0})}\multimap_{\mathcal{E}}}{\lambda X.\mathit{have}(X,Y) : e(\boxed{1}) \multimap t(\boxed{0})}\multimap_{\mathcal{I},1} \quad \begin{array}{l}\lambda S.\mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, S(Z)) : \\ \forall H.[e(\boxed{1}) \multimap t(H)] \multimap t(H)\end{array}}{\dfrac{\mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,Y)) : t(\boxed{0})}{\lambda Y.\mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,Y)) : e(\boxed{3}) \multimap t(\boxed{0})}\multimap_{\mathcal{I},2}}\multimap_{\mathcal{E}}$$

(80)
$$\dfrac{\begin{array}{l}\lambda Y.\mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,Y)) : \\ e(\boxed{3}) \multimap t(\boxed{0})\end{array} \quad \begin{array}{l}\lambda S.\lambda Y.\mathit{all}(X, |X| = 1 \wedge X \subset Y, S(X)) : \\ \forall G,H.\ [e(G) \multimap t(H)] \multimap [e(G) \multimap t(H)]\end{array}}{\begin{array}{l}\lambda Y.\mathit{all}(X, |X| = 1 \wedge X \subset Y, \mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,X))) : \\ e(\boxed{3}) \multimap t(\boxed{0})\end{array}}\multimap_{\mathcal{E}}$$

(81)
$$\dfrac{\begin{array}{l}\lambda Y.\mathit{all}(X, |X| = 1 \wedge X \subset Y, \\ \mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,X))) : \\ e(\boxed{3}) \multimap t(\boxed{0})\end{array} \quad \begin{array}{l}\lambda S.\mathit{exists}(Y, |Y| = 2 \wedge \mathit{tattoo}^s(Y), S(Y)) : \\ \forall H.[e(\boxed{3}) \multimap t(H)] \multimap t(H)\end{array}}{\begin{array}{l}\mathit{exists}(Y, |Y| = 2 \wedge \mathit{tattoo}^s(Y), \\ \quad \mathit{all}(X, |X| = 1 \wedge X \subset Y, \\ \qquad \mathit{exists}(Z, \mathit{boy}^s(Z) \wedge |Z| > 1, \mathit{have}(Z,X)))) : t(\boxed{0})\end{array}}\multimap_{\mathcal{E}}$$

A preliminary solution to this problem – presented in greater detail and further refined in Przepiórkowski 2014a – is inspired by the Glue Semantics approach to Negative Polarity Licensing proposed by Fry 1999. The original intuition behind this approach is that a Negative Polarity Item (NPI) "attaches" to its usual meaning a marker which is transferred during the semantic derivation until it meets a licensor which discharges (i.e. consumes) it. In the case at hand, the distributive share acts as an NPI and the marker is discharged when the distributive meaning of *po* combines with a meaning containing the contribution of this distributive share.

Technically, we introduce the "marked" type t^d, modify the distributive meaning constructor so that it eliminates the marking (we will call it **[distr-E]**), and add another meaning constructor in the lexical entry of *po* which introduces the marking (we will call it **[distr-I]**); compare the lexical entry (82) for *po* below with (40) above:

(82) *po* P $(\uparrow \text{PRED}) = \text{‘PO<OBJ>’}$
[po] = $\lambda P.P : \forall F.\ [e(\uparrow) \multimap t(F)] \multimap [e((\uparrow \text{OBJ})) \multimap t(F)]$
[distr-E] = $\lambda S.\lambda Z.all(X, |X| = 1 \wedge X \subset Z, S(X)) :$
$\forall G, H.\ [e(G) \multimap t^d(H)] \multimap [e(G) \multimap t(H)]$
[distr-I] = $\lambda Q.Q :$
$\forall H.\ [[e((\uparrow \text{OBJ})) \multimap t(H)] \multimap t(H)] \multimap [[e((\uparrow \text{OBJ})) \multimap t(H)] \multimap t^d(H)]$

In the running example, given the f-structure (41), the three meaning constructors in the lexical entry of *po* instantiate to:

(83) **[po]**
$\lambda P.P : \forall F.[e(\boxed{2}) \multimap t(F)] \multimap [e(\boxed{3}) \multimap t(F)]$

(84) **[distr-E]**
$\lambda S.\lambda Z.all(X, |X| = 1 \wedge X \subset Z, S(X)) : \forall G, H.\ [e(G) \multimap t^d(H)] \multimap [e(G) \multimap t(H)]$

(85) **[distr-I]**
$\lambda Q.Q : \forall H.\ [[e(\boxed{3}) \multimap t(H)] \multimap t(H)] \multimap [[e(\boxed{3}) \multimap t(H)] \multimap t^d(H)]$

With these meaning constructors, the proof of the correct meaning in the running example is similar to that in (55), with **[distr]** replaced by **[distr-E]** and with **[distr-I]** combining with the meaning of *dwa tatuaże* ‘two tattoos’. Modified partial conclusions are presented below (unchanged constructors are repeated for convenience):

(45′) **[have-po]**
$\lambda X.\lambda Y.have(X, Y) : e(\boxed{1}) \multimap [e(\boxed{3}) \multimap t(\boxed{0})]$

(47′) **[two-tattoos]**
$\lambda S.exists(Y, |Y| = 2 \wedge tattoo^s(Y), S(Y)) : \forall H.\ [e(\boxed{3}) \multimap t(H)] \multimap t(H)$

(86) **[distr-I-two-tattoos]**
$\lambda S.exists(Y, |Y| = 2 \wedge tattoo^s(Y), S(Y)) : \forall H.\ [e(\boxed{3}) \multimap t(H)] \multimap t^d(H)$

(87) **[have-po-distr-I-two-tattoos]**
$\lambda X.exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X, Y)) : e(\boxed{1}) \multimap t^d(\boxed{0})$

(88) **[distr-E-have-po-distr-I-two-tattoos]** (= **[distr-have-po-two-tattoos]** in proof (55))
$\lambda Z.all(X, |X| = 1 \wedge X \subset Z, exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X, Y))) : e(\boxed{1}) \multimap t(\boxed{0})$

(89) **[boys-existential]** (= conclusion in subproof (53) = **[boys-existential]** in proof (55))
$\lambda S.exists(Z, boy^s(Z) \wedge |Z| > 1, S(Z)) : \forall H.[e(\boxed{1}) \multimap t(H)] \multimap t(H)$

(90) **[boys-existential-distr-E-have-po-distr-I-two-tattoos]** (= conclusion in proof (55))
$exists(Z, boy^s(Z) \wedge |Z| > 1,$
$all(X, |X| = 1 \wedge X \subset Z,$
$exists(Y, |Y| = 2 \wedge tattoo^s(Y), have(X, Y)))) : t(\boxed{0})$

Note how the marking ^{d} is introduced by **[distr-I]** on the quantifier *two tattoos* in (86), how it is transferred to the predicate in (87) and how it is eliminated by **[distr-E]**, which now expects its semantic argument to be so marked, in (88). The proof is summarised below.

(91)

$$\dfrac{\dfrac{\dfrac{\dfrac{[\textbf{have}][\textbf{po}]}{[\textbf{have-po}]}\multimap_{\mathcal{E}\mathcal{E}I} \quad \dfrac{\dfrac{[\textbf{two}][\textbf{tattoos}]}{[\textbf{two-tattoos}]}\multimap_{\mathcal{E}} \quad [\textbf{distr-I}]}{[\textbf{distr-I-two-tattoos}]}\multimap_{\mathcal{E}}}{[\textbf{have-po-distr-I-two-tattoos}]}\multimap_{\mathcal{E}\mathcal{E}I} \quad [\textbf{distr-E}]}{[\textbf{distr-E-have-po-distr-I-two-tattoos}]}\multimap_{\mathcal{E}} \quad \dfrac{[\textbf{boys}][\textbf{existential}]}{[\textbf{boys-existential}]}\multimap_{\mathcal{E}}}{[\textbf{boys-existential-distr-E-have-po-distr-I-two-tattoos}]}\multimap_{\mathcal{E}}$$

At the same time, the unwanted proof (78) for the same sentence (26) is blocked now. Since the constructor **[distr-I]** may only combine with the constructor of a quantifier whose restriction is expressed by the object of *po*, it cannot combine with the existential *chłopcy* 'boys', whose restriction on the glue side contains $e(\boxed{1})$ instead of the $e(\boxed{3})$ expected by **[distr-I]**. Hence, **[boys-existential]** in a putative analogue of proof (78) cannot contain the marker $d(\ldots)$, so it cannot pass it to **[boys-existential-have-po]**, and so **[distr-E]** cannot combine with it. While **[distr-I]** may still combine with **[two-tattoos]**, neither the resulting **[distr-I-two-tattoos]** nor **[distr-E]** may enter the proof now.

5 Conclusion

Analyses of distance distributivity, such as Choe 1987, Safir and Stowell 1988, Moltmann 1997, Zimmermann 2002 or Dotlačil 2012, have so far been formulated mainly within the transformational paradigm. In contrast, the current paper provides a non-transformational analysis, couched within Lexical Functional Grammar and coupled with the morphosyntactic account of Przepiórkowski and Patejuk 2013. On the semantic side, we employed the resource-sensitive approach of Glue Semantics. Empirically, the main point of this paper is the introduction – and successful analysis – of a construction troublesome for previous analyses, where the sorting key is syntactically embedded in the phrase expressing the distributive share.

The account proposed here is still at a relatively early stage of development. It remains to be seen whether the mechanism employed to harness overgeneration, introduced in section 4.4, is sufficiently general and robust. Moreover, we had nothing to say about distribution over events, witnessed in Polish and German, among other languages. (Both points are addressed in Przepiórkowski 2014a.) Nevertheless, we hope that the current proposal provides a reasonable backbone to flesh out a more exhaustive constraint-based and resource-sensitive analysis of distance distributivity in Polish and other languages.

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