Categorematic Unreducible Polyadic Quantifiers in Lexical Resource Semantics

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Introduction

(1) Two agencies in my country spy on different citizens.

(2) a. Two agencies in my country spy on different citizens from the ones we know.
b. Two agencies in my country spy on various/many citizens.
c. The citizens that one of the agencies spies on are different from the citizens that the other agency spies on.

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Analogous Sentences

(3) a. Two agencies in my country spy on different citizens.
b. Three agencies in my country spy on different citizens.
c. Four agencies in my country spy on different citizens.
d. …
e. Every agency in my country spies on different citizens.
f. All agencies in my country spy on different citizens.
g. Many agencies in my country spy on different citizens.
h. Most agencies in my country spy on different citizens.
i. …

Overview

Outline

Types of Quantifiers

Quantifier	Lindström type	Functional type
(two agencies)	$\langle 1 \rangle$	$\langle\langle et\rangle\, t\rangle$
(two agencies, all citizens)	$\langle 2 \rangle$	$\langle\langle e\,\langle et\rangle\rangle\, t\rangle$
(two teachers, every girl, many books)	$\langle 3 \rangle$	$\langle\langle e\,\langle e\,\langle et\rangle\rangle\rangle\, t\rangle$
($NP_1, \ldots, NP_n$)	$\langle n \rangle$	$\ldots$
(two)	$\langle 1, 1 \rangle$	$\langle\langle et\rangle\,\langle\langle et\rangle\, t\rangle\rangle$
(two, all)	$\langle 1^2, 2 \rangle = \langle 1, 1, 2 \rangle$	$\langle\langle et\rangle\,\langle\langle et\rangle\,\langle\langle e\,\langle et\rangle\rangle\, t\rangle\rangle\rangle$
(two, every, many)	$\langle 1^3, 3 \rangle = \langle 1, 1, 1, 3 \rangle$	$\langle\langle et\rangle\,\langle\langle et\rangle\,\langle\langle et\rangle\,\langle\langle e\,\langle e\,\langle et\rangle\rangle\rangle\, t\rangle\rangle\rangle\rangle$
…	…	…

Examples

(4) $[_{NP}$ Most apes$]$ $[_{VP}$ picked $[_{NP}$ ten berries$]]$.

(5) a. most $x(\text{ape}(x) : (10\ y(\text{berry}(y) : \text{pick}(x, y))))$
b. most (ape : $(\lambda x.10$ (berry : $\lambda y.\text{pick}(x, y))))$
c. (most apes , 10 berries) pick
d. (most, 10) (apes, berries, pick)

Fregean quantifiers of type $\langle 2 \rangle$ are reducible in the sense that they can be thought of as being composed of two iterated type $\langle 1 \rangle$ quantifiers: They result from function composition of two type $\langle 1 \rangle$ quantifiers:

(6) (most apes , 10 berries) pick
$=$ (most apes) $\circ$ (10 berries) pick
$=$ (most apes)((10 berries) pick)

Background and Notation

Definition (unary quantifiers as 1-ary relation reducers)
Assume a universe E, and for each integer n a relation $R \subseteq E^{n+1}$ and an $\langle 1 \rangle$-ary quantifier Q.

$$Q(R) :=$$
$$\{(x_1, \ldots, x_n) \in E^n |$$
$$Q(\{y_1 \in E | (x_1, \ldots, x_n, y_1) \in R\}) = 1\}$$

Notational Convention
Given a set E and a binary relation R, $R \subseteq E^2$, for each $x \in E$, we write Rx for the set of objects x bears R to: $Rx = \{y | (x, y) \in R\}$.

Example:
The set of berries ape a picks: $\text{pick}\, a = \{b | (a, b) \in \text{pick}\}$

Outline

A Semantics for Quantifiers with “different”

Definition: Semantics of a quantifier containing DIFFERENT (Δ) (adapted from Keenan and Westerståhl (1997))

For $\mathcal{Q}$ a polyadic quantifier of type $\langle 1^2, 2\rangle$ containing Δ, $A, B \subseteq E$, $R \subseteq E^2$, and Q a quantifier of type $\langle 1, 1\rangle$, the interpretation of $\mathcal{Q}$ is as follows:
$\mathcal{Q}(A, B, R) = 1$ iff there is an A', $A' \subseteq A$ such that

$Q(A, A') = 1$, *and*
for all $x, y \in A'$: $(x \neq y) \Rightarrow (B \cap Rx \neq B \cap Ry)$.

(7) a. [$_{NP}$ Two apes] picked [$_{NP}$ different berries].
b. (two, different) (apes, berries, pick)
c. #(two apes, different berries) pick

Unreducibility

Definition: (Reducibility, Dekker 2003)
A type $\langle 2 \rangle$ quantifier Q is $\langle 2 \rangle$-reducible iff there are two type $\langle 1 \rangle$ quantifiers Q_1 and Q_2 with $Q = Q_1 \circ Q_2$.

Theorem: (Reducibility Equivalence, Keenan 1992)
For every domain E and Q_1 and Q_2 reducible quantifiers of type $\langle 2 \rangle$:

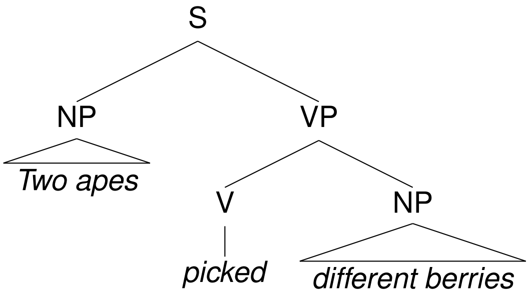
$$Q_1 = Q_2 \textit{ iff for all } A, B \subseteq E\textit{:} \quad Q_1(A \times B) = Q_2(A \times B)$$

Lemma:
Assume a universe E with at least two elements, $A, B \subseteq E$, a standard definition of the type $\langle 1, 1 \rangle$ quantifier 'two', and the semantics for quantifiers with *different* as shown above.
Then (two A, different B) is unreducible.

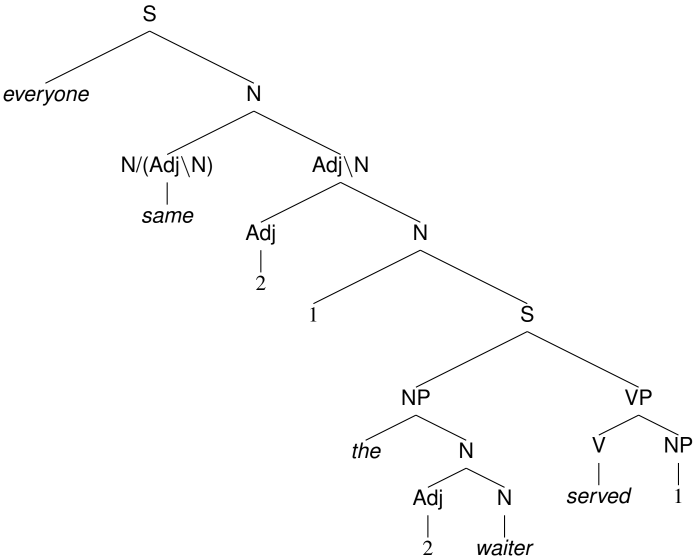
Outline

Interim Summary

1. It is impossible to interpret the two NPs *two apes* and *different berries* independently as generalized quantifiers and obtain a semantics as stated above.
2. A standard compositional semantic analysis with the assumed meaning is impossible with the syntactic structure provided by a standard HPSG analysis:



Other Theories: *The same waiter served everyone*



(adapted from Barker (2007))

A Reducible Polyadic Quantifier in LRS

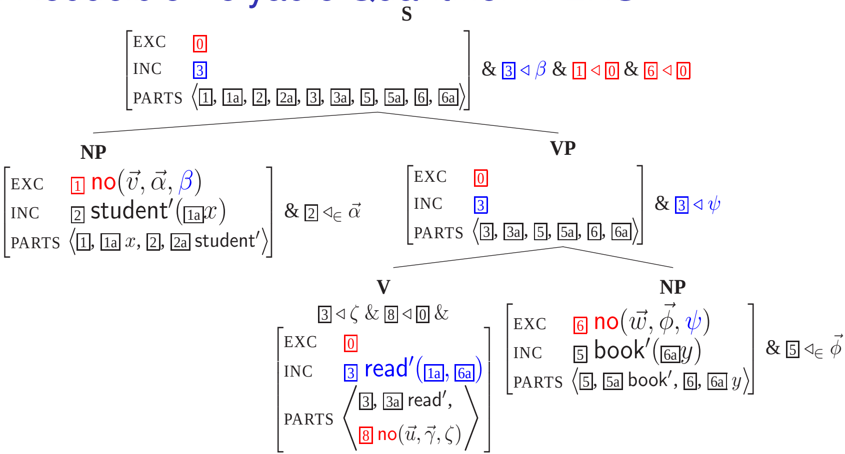


Figure 1: LRS analysis of *Niciun student nu a citit nicio carte*

Available interpretations:

a. $no(x, student(x), no(y, book(y), read(x,y)))$: $\boxed{0} = \boxed{1} \wedge \boxed{6} \triangleleft \beta$ **[DN]**

b. $no((x,y), (student(x), book(y)), read(x,y))$: $\boxed{0} = \boxed{1} = \boxed{6}$ **[NC]**

Outline

Categorematic Quantifiers in LRS (1)

two as a type (1,1) quantifier:

$$\begin{bmatrix} \text{PHON} & \langle \textit{two} \rangle \\ \text{SS LOC CONT} & \begin{bmatrix} \text{INDEX DR} & x \\ \text{MAIN} & \text{two}' \end{bmatrix} \\ \text{LRS} & \begin{bmatrix} \text{EXC} & \textit{me} \\ \text{INC} & \boxed{4}\ \text{two}'(\lambda x.\alpha, \lambda x.\beta) \\ \text{PARTS} & \langle \boxed{4}, \boxed{4a}\ x, \boxed{4b}\ \text{two}', \boxed{4c}(\lambda x.\alpha), \boxed{4d}\ (\lambda x.\beta), \boxed{4e}\ \text{two}'(\lambda x.\alpha) \rangle \end{bmatrix} \end{bmatrix}$$

$\&\ x \triangleleft \alpha\ \&\ x \triangleleft \beta$

(to be generalized below)

Categorematic Quantifiers in LRS (2)

$$\begin{bmatrix} \text{PHON} & \langle \textit{two, agencies} \rangle \\ \text{SS LOC CONT INDEX DR} & x \\ \text{LRS} & \begin{bmatrix} \text{EXC} & \boxed{4}\ \text{two}'(\lambda x.\alpha, \lambda x.\beta) \\ \text{INC} & \boxed{5}\ \text{agency}'(\boxed{4a}x) \\ \text{PARTS} & \left\langle \begin{array}{l} \boxed{4}, \boxed{4a}\ x, \boxed{4b}\ \text{two}', \boxed{4c}\ (\lambda x.\alpha), \boxed{4d}\ (\lambda x.\beta), \\ \boxed{4e}\ \text{two}'(\lambda x.\alpha), \boxed{5}, \boxed{5a}\text{agency}' \end{array} \right\rangle \end{bmatrix} \end{bmatrix}$$

$\&\ \boxed{5} \triangleleft \alpha$

$\&\ x \triangleleft \alpha$

$\&\ x \triangleleft \beta$

(to be generalized below)

Semantics of a quantifier containing DIFFERENT

(for languages of Ty2 with polyadic quantifiers)

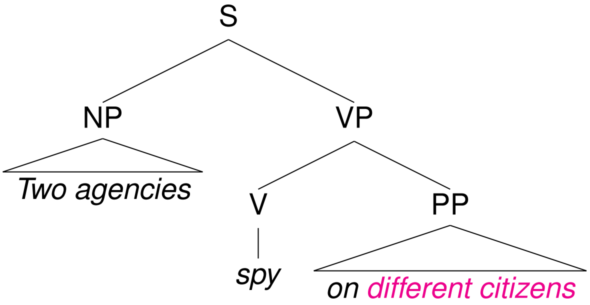
Definition:
For $\mathcal{Q} = (Q, \Delta)$ a polyadic quantifier of type $\langle 1^2, 2 \rangle$ containing Δ, x, y variables of type e, α, β expressions of type t, Q a monadic generalized quantifier, and ρ an expression of type $\langle e \langle et \rangle \rangle$, the interpretation of $\mathcal{Q}$ is as follows:

$[\![(Q, \Delta)(\lambda x.\alpha, \lambda y.\beta, \rho)]\!]^{M,g} = 1$ iff there is an A', $A' \subseteq [\![\lambda x.\alpha]\!]^{M,g}$, such that

$$[\![Q(\lambda x.\alpha)]\!]^{M,g}(A') = 1, \textit{ and}$$

$$\textit{for all } e_1, e_2 \in A': \quad e_1 \neq e_2 \Rightarrow$$
$$[\![\lambda y.\beta]\!]^{M,g} \cap [\![\rho]\!]^{M,g}(e_1) \neq [\![\lambda y.\beta]\!]^{M,g} \cap [\![\rho]\!]^{M,g}(e_2).$$

Two agencies spy on different citizens (1)



Two agencies spy on different citizens (2)

$$
\left[\begin{array}{ll}
\text{PHON} & \langle \textit{different} \rangle \\
\text{SS LOC} & \left[\begin{array}{ll}
\text{CAT HD SPEC} \ldots & \left[\begin{array}{ll} \text{INDEX DR} & y \\ \text{MAIN} & \zeta \end{array}\right] \\
\text{CONT} & \left[\begin{array}{ll} \text{INDEX DR} & y \\ \text{MAIN} & \Delta \end{array}\right]
\end{array}\right] \\
\text{LRS} & \left[\begin{array}{ll}
\text{EXC} & \textit{me} \\
\text{INC} & \boxed{1}\ {\color{red}(\gamma, \Delta)(\sigma_1, \lambda y.\beta, \ldots \lambda y.\rho)} \\
\text{PARTS} & \left\langle \begin{array}{l} \boxed{1}, \boxed{1a}\ y, \boxed{1b}\ \Delta, \boxed{1c}\ (\gamma, \Delta), \boxed{1d}(\lambda y.\beta), \boxed{1e}\ (\lambda y.\rho), \\ \boxed{1f}\ (\gamma, \Delta)(\sigma_1), \boxed{1g}\ (\gamma, \Delta)(\sigma_1, \lambda y.\beta) \end{array} \right\rangle
\end{array}\right]
\end{array}\right]
$$

$$\& \ y \triangleleft \beta \ \& \ y \triangleleft \rho \ \& \ {\color{blue}\zeta \triangleleft \beta}$$

Two agencies spy on different citizens (3)

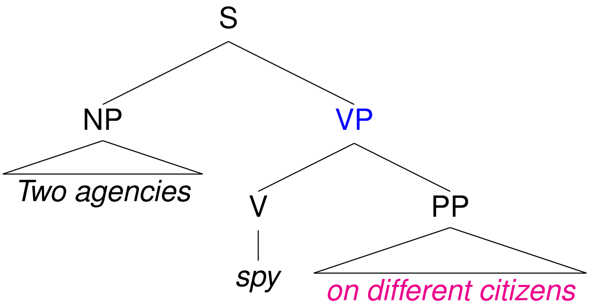
$$\begin{bmatrix} \text{PHON} & \langle \textit{citizens} \rangle \\ \text{SS LOC CONT} & \begin{bmatrix} \text{INDEX DR} & \boxed{1a}\ \textit{var} \\ \text{MAIN} & \boxed{2a}\ \text{citizen}' \end{bmatrix} \\ \text{LRS} & \begin{bmatrix} \text{EXC} & \textit{me} \\ \text{INC} & \boxed{2}\ \text{citizen}'(\boxed{1a}) \\ \text{PARTS} & \langle \boxed{2}, \boxed{2a}\ \text{citizen}' \rangle \end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} \text{PHON} & \langle \textit{different, citizens} \rangle \\ \text{SS LOC CONT} & \begin{bmatrix} \text{INDEX DR} & \boxed{1a}\ y \\ \text{MAIN} & \boxed{2a}\ \text{citizen}' \end{bmatrix} \\ \text{LRS} & \begin{bmatrix} \text{EXC} & \boxed{1}\ (\gamma, \Delta)(\sigma_1, \lambda y.\beta, \ldots \lambda y.\rho) \\ \text{INC} & \boxed{2}\ \text{citizen}'(\boxed{1a} y) \\ \text{PARTS} & \left\langle \begin{matrix} \boxed{1}, \boxed{1a}\ y, \boxed{1b}\ \Delta, \boxed{1c}\ (\gamma, \Delta), \\ \boxed{1d}\ (\lambda y.\beta), \boxed{1e}\ (\lambda y.\rho), \boxed{1f}\ (\gamma, \Delta)(\sigma_1), \\ \boxed{1g}\ (\gamma, \Delta)(\sigma_1, \lambda y.\beta), \boxed{2}, \boxed{2a}\ \text{citizen}' \end{matrix} \right\rangle \end{bmatrix} \end{bmatrix} \;\&\; \boxed{2a} \triangleleft \beta \;\&\; y \triangleleft \rho$$

Two agencies spy on different citizens (4)

1. SEMANTICS PRINCIPLE, clause on *head-adjunct* structures: In a *head-adjunct-phrase*, the EXCONT of the non-head is a component of the EXCONT of the head (Richter & Sailer, 2003).
2. Given that (i) *different citizens* is the maximal projection of the noun *citizens*, and (ii) *different* does not project, it follows that the INCONT of *different* is the EXCONT of *different citizens*.
3. Lexical restrictions: *different* requires that its DR value (variable y) be identical with the DR value of *citizens*, and the MAIN value of *citizens* be a component of the restrictor corresponding to Δ.
4. Other relevant LRS Principles: PROJECTION PRINCIPLE, CONTENT PRINCIPLE, clause of SEMANTICS PRINCIPLE for quantifier/nominal head combinations.
5. New subclauses of the head-adjunct clause of Richter & Sailer's (2003) LRS specification might be necessary in larger noun phrases.

Two agencies spy on different citizens (5)



- PP *on different citizens*:
  on as case marking preposition is a semantic content raiser
- VP *spy on different citizens*:
  regular quantifier/verb phrase projection combination

Two agencies spy on different citizens (6)

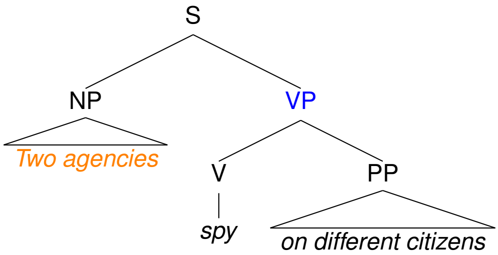
$$
\left[\begin{array}{ll}
\text{PHON} & \langle \textit{spy, on, different, citizens} \rangle \\
\text{SS LOC CONT MAIN} & \boxed{3b}\ \text{spy}' \\
\text{LRS} & \left[\begin{array}{ll}
\text{EXC} & \boxed{0} \\
\text{INC} & \boxed{3}\ \text{spy}'(\boxed{4a}, \boxed{1a}\ y) \\
\text{PARTS} & \left\langle \begin{array}{l}
\boxed{3}, \boxed{3a}\ \text{spy}'(\boxed{1a} y), \boxed{3b}\ \text{spy}', \\
\boxed{1}, \boxed{1a}\ y, \boxed{1b}\ \Delta, \boxed{1c}\ (\gamma, \Delta), \\
\boxed{1d}\ (\lambda y.\beta), \boxed{1e}\ (\lambda y.\rho), \boxed{1f}\ (\gamma, \Delta)(\sigma_1), \\
\boxed{1g}\ (\gamma, \Delta)(\sigma_1, \lambda y.\beta), \boxed{2}, \boxed{2a}\ \text{citizen}'
\end{array} \right\rangle
\end{array}\right]
\end{array}\right]
$$

$\& \ \boxed{2a} \triangleleft \beta$

$\& \ \boxed{3} \triangleleft \rho$

$\& \ \boxed{1}\ (\gamma, \Delta)(\sigma_1, \lambda y.\beta, \ldots \lambda y.\rho) \triangleleft \boxed{0}$

Two agencies spy on different citizens (7)



Insight:

VP semantics: $(\gamma, \Delta)(\sigma_1, \lambda y.\text{citizen}'(y), \ldots \lambda y.\text{spy}'(\boxed{4a}, \boxed{1a}\ y))$ and
NP semantics: $\text{two}'(\lambda x.\text{agency}'(x), \lambda x.\beta)$
become compatible if NP semantics is underspecified, essentially following the Negative Concord idea by Iordăchioaia & Richter (2015):

NP semantics: $(\text{two}', \psi)(\lambda x.\text{agency}'(x), \sigma_2, \lambda x.\kappa)$

Two agencies spy on different citizens (8)

$$\left[\begin{array}{ll}\text{PHON} & \langle \textit{two, agencies} \rangle \\ \text{SS LOC CONT INDEX DR} & x \\ \text{LRS} & \left[\begin{array}{ll}\text{EXC} & \boxed{4}\ (\text{two}', \psi)(\lambda x.\alpha, \sigma_2, \lambda x.\kappa) \\ \text{INC} & \boxed{5}\ \text{agency}'(\boxed{4a}\,x) \\ \text{PARTS} & \left\langle \begin{array}{l}\boxed{4}, \boxed{4a}\ x, \boxed{4b}\ \text{two}', \boxed{4c}\ (\text{two}', \psi), \boxed{4d}\ (\lambda x.\alpha), \boxed{4e}\ (\lambda x.\kappa), \\ \boxed{4f}\ (\text{two}', \psi)(\lambda x.\alpha), \boxed{4g}\ (\text{two}', \psi)(\lambda x.\alpha, \sigma_2), \boxed{5}, \boxed{5a}\,\text{agency}' \end{array}\right\rangle \end{array}\right] \end{array}\right]$$

$$\& \ \boxed{5} \triangleleft \alpha \ \& \ x \triangleleft \kappa$$

- *two*'s INCONT (identical to $\boxed{4}$ above) is underspecified to have a functor of type $\langle 1^n, n \rangle$.
- For a well-formed Ty2 expression in the EXCONT of our sentence, its type must be $\langle 1^2, 2 \rangle$.
- *two agencies* as shown above regularly combines with *spy on different citizens* (quantifier/verb phrase projection combination).

Two agencies spy on different citizens (9)

$$\begin{bmatrix} \text{PHON} & \langle \textit{two, agencies, spy, on, different, citizens} \rangle \\ \text{SS LOC CONT MAIN} & \text{spy}' \\ \text{LRS} & \begin{bmatrix} \text{EXC} & (\text{two}', \Delta)(\lambda x.\text{agency}'(x), \lambda y.\text{citizen}'(y), \lambda x \lambda y.\text{spy}'(x, y)) \\ \text{INC} & \text{spy}'(x, y) \\ \text{PARTS} & \langle \ldots \rangle \end{bmatrix} \end{bmatrix}$$

The EXCONT above is the only way to resolve the restrictions imposed on possible well-formed combinations of the lexically contributed logical expressions.

Outline

Quick overview: *different*'s Environments

- The analysis covered: quantified NP + plural different NP
  also: quantified NP in other configurations, scope-dependent
- quantified NP + Det plural different NP
  Every ape picked three different berries.
  $(\forall, (3, \Delta)))(\lambda x.\text{ape}'(x), \lambda y.\text{berry}'(y), \lambda x \lambda y.\text{pick}'(x, y))$
- quantified NP + singular different: ?
  Every boy recited a different poem.
- No NP + plural different NP: decomposition?
  No boy recited different poems.
- plural NP / NP conjunction + different NP
- Every boy claimed that every girl read a different poem.
  (Bumford & Barker 2013)
- Plural events, reciprocal reading, external reading(s)
  esp. Beck (2000), Brasoveanu (2011)
- John and Mary want to live in different cities. (David Lahm, pc)

Insights

- Keenan's semantics of polyadic quantifiers with *different* can be combined with an HPSG syntax.
- It might be worth pursuing…
- …although the proposal is behind the competition in terms of coverage of constructions involving *different*.
- LRS provides all that is necessary to avoid
  - fancy LF syntax
  - appealing to pragmatics for fixing a weaker semantics
- Analysis could feed the Ty2 inferencing architecture of Hahn & Richter (2015)

Thank you!

n-ary Generalized Quantifiers

Definition (n-ary quantifiers as n-ary relation reducers)
Assume a universe E, and for each integer m, n (with $n \geq 1$) a relation $R \subseteq E^{m+n}$ and an $\langle n \rangle$-ary quantifier Q.

$$Q(R) := \{(x_1, \ldots, x_m) \in E^m | \quad Q(\{(y_1, \ldots, y_n) \in E^n | (x_1, \ldots, x_m, y_1, \ldots, y_n) \in R\}) = 1\}$$

Example:
Assume a binary relation ‘spy’ and a world in which two agencies, ‘nsa’ and ‘cia’, spy on every citizen c_k. Let Q be the binary quantifier (two agencies, every citizen).

$$Q(\text{spy}) = \{() \in E^0 | Q(\{(y_1, y_2) \in E^2 | (y_1, y_2) \in \text{spy}\}) = 1\}$$

Since for all citizens c_k, $(\text{nsa}, c_k) \in$ spy and $(\text{cia}, c_k) \in$ spy, we obtain $Q(\text{spy}) = \{()\} = 1$

Cartesian Product Relations

Assume $E = \{a, c\}$. Thus, $E^2 = \{(a, a), (a, c), (c, a), (c, c)\}$.

Cartesian product relations on E:
$(\mathcal{P}(E) \times \mathcal{P}(E))$

$\{\{\}, \{(a, a)\}, \{(a, c)\}, \{(c, a)\}, \{(c, c)\},$
$\{(a, a), (a, c)\}, \{(c, a), (c, c)\}, \{(a, a), (c, a)\}, \{(a, c), (c, c)\},$
$\{(a, a), (a, c), (c, a), (c, c)\}\}$

Other binary relations on E not among the Cartesian product relations:
(but in $\mathcal{P}(E^2)$)

$\{(a, a), (c, c)\}$, $\{(a, c), (c, a)\}$, and $\{(a, a), (a, c), (c, a)\}$, among others.

Unreducibility of $(2, \Delta)^{A,C}$

Let 0 be a unary quantifier that is false on all unary relations. Then $0 \circ 0$ is a reducible type $\langle 2 \rangle$ quantifier that is false on all binary relations.

Assume a world with two agencies $A = \{a_1, a_2\}$ and two citizens $C = \{c_1, c_2\}$.

Assume $R \subseteq A \times C$.
$(2, \Delta)^{A,C}(R) = 0$ for all $R = A \times C$ with $|A| < 2$ or $|C| < 2$.
Likewise for $0 \circ 0(R)$.

Finally, assume $R = A \times C = \{(a_1, c_1), (a_1, c_2), (a_2, c_1), (a_2, c_2)\}$.
$\Rightarrow$ $(2, \Delta)^{A,C}(R) = 0$; and, of course, $0 \circ 0(R) = 0$
$\Rightarrow$ Keenan's theorem: If $(2, \Delta)^{A,C}$ reducible, then $(2, \Delta)^{A,C} = 0 \circ 0$.

However, note that for $R = \{(a_1, c_1), (a_2, c_2)\}$, we get $(2, \Delta)^{A,C}(R) = 1$, whereas of course $0 \circ 0(R) = 0$.

$\Rightarrow$ $(2, \Delta)^{A,C}$ is unreducible qed